

**COT 3420
SPRING 2007
Section U2**

EXAM # 2

INSTRUCTIONS

1. The exam is open book, open notebook.
2. There are 7 questions on the test, for a total of 100 points.
3. For Question 1, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour and 15 minutes to work on the test.
5. Write the answers to questions 1, 3, 4, 5, and 6 on the exam paper. Write the other answers on the blank sheets.
6. Print your name below.

NAME:

QUESTIONS

Question 1. (27 points)

The universe of structure \mathcal{A} is $\{3, 4, 5, 6\}$. The \mathcal{A} interpretations of a , x , and y are $a^{\mathcal{A}} = 3$, $x^{\mathcal{A}} = 5$, $y^{\mathcal{A}} = 6$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicate $P^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[f(f(x))]$ =
2. $\mathcal{A}[g(x, y)]$ =
3. $\mathcal{A}[g(g(a, y), f(x))]$ =
4. $\mathcal{A}[P(x, f(a))]$ =

Question 4 (8 points)

Skolemize the formula

$$F = \forall x \exists y \forall z \exists u \forall v \exists w [((P(x, y) \vee P(a, f(z))) \wedge \neg P(h(v, x), b)) \wedge (P(u, f(v)) \vee \neg P(v, w))].$$

Don't show your work, just write the Skolemized formula.

Write the substitutions as [variable/term]. Display the answer below.

Question 5 (10 points)

Find a prenex form for

$$F = [\forall x (\forall y P(x, y) \vee \exists z Q(x, z)) \wedge \forall u (\exists v \neg P(u, v) \vee \exists w \neg Q(w, u))].$$

Don't show your work, just write the prenex form. Display your answer below.

Question 6 (3 points)

Find an existential closure of the formula $F = \forall u \exists y P(x, y, z, u, v)$.

Write your answer below.

Question 7 (20 points)

Part 1. (8 points)

Show that when t is free for x in F , $\forall x F \models F[x/t]$

Hint: Apply the translation lemma.

Part 2. (12 points)

Show that when t is not free for x in F , $\forall x F \models F[x/t]$ is not always true.

Hint: Construct a counter-example.