COT 3420
Sections U1
SPRING 2007

FINAL EXAM

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 10 questions on the test, for a total of 176 points.
3. If you took the induction retest, I will add 9 points to the exam score.
4. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 2 hours and a half to complete the exam.
7. Mark the answers to questions 2,3,4,5,6, and 8 on the exam paper. Write the answers to the other questions on the blank sheets.
8. No talking to each other during the test!
9. Write your name and panther id below.

NAME: _
Panther ID:_ _

QUESTIONS

Question 1. (25 points)

Let C be a clause and $L_1, L_2, \ldots L_n$, $n \geq 2$, be a set of literals of C such that $S = \{L_1, \ldots, L_n\}$ is unifiable. Let σ be a mgu of S. The clause $F = \sigma[C]$ is called a **factor** of C.

For example, let $C = \{P(x,y), P(y,z), Q(x,y,z)\}$ and $S = \{P(x,y), P(y,z)\}$. S is unifiable and $\sigma = [y/x, z/x]$ is a mgu of S. Then, the clause $R = \sigma[C] = \{P(x,x), Q(x,x,x)\}$ is a factor of C.

Now, let C be a clause, R a factor of C, and $\bar{C}$ and $\bar{R}$ be the universal closure of C and R.

Prove that $\bar{C} \models \bar{R}$.

Write your proof on a blank sheet of paper.

Question 2. (20 points)

Write the predicate partition(Pivot,List,List1,List2) that splits List into List1 and List2 in such a way that List1 contains the elements of List that are less than or equal to Pivot and List2 contains the items greater than Pivot. For example partition(12,[3, 20, 23, 5, 12, 18, 4 ,6], L1,L2) is satisfied if L1 = [3, 5, 12, 4, 6] and L2 = [20, 23, 18].

Write your answer below.

Question 3. (20 points)

Write $D[F,2]$ for $F = \forall x \forall y [P(f(x,y),y) \wedge (\neg P(x,g(x,y)) \vee \neg P(f(x,x),y))]$. Write your answer below.

Question 4. (10 points)

Write all resolvents of the clauses $C_1 = \{P(x, f(x), y), P(g(y), z, h(u)), Q(x, y, z)\}$ and $C_2 = \{\neg P(x, y, z), Q(x, z, y)\}$. Don't write the relabelings, the unification set, and the mgu, just display the resolvents. Write your answer below.

Question 5. (10 points)

Find out if $S = \{P(f(x), y, h(x, z)), P(y, f(g(u)), v), P(f(g(a)), f(w), h(w, f(w)))\}$ is unifiable. If so, display a mgu; otherwise write that S is not unifiable. Write your answer below.

Question 6. (30 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. $[x/z, y/x, v/w] \diamond [x/y, y/u, u/x, w/v] = \ldots$
   a. $[y/u, u/z]$
   b. $[x/z, u/x, w/v]$
   c. $[y/u, u/z, v/w]$
   d. $[x/z, y/u, u/z, v/w]$
2. $[v/b] \diamond [z/h(u)] \diamond [x/g(v)] \diamond [y/f(x,z)] = \ldots$
   a. $[y/f(g(v),z), x/g(v), z/h(u), v/b]$
   b. $[y/f(g(v),h(u)), x/g(b), z/h(b), v/b]$
   c. $[y/f(g(b),h(u)), x/g(v), z/h(u), v/b]$
   d. $[y/f(g(b),h(u)), x/g(b), z/h(u), v/b]$
3. ... is unifiable.
   a. $\{P(a,x), P(x,b)\}$
   b. $\{P(x,f(y)), P(x,a)\}$
   c. $\{P(x,y), P(f(x),z)\}$.
   d. $\{P(f(x),x), P(y,g(z))\}$.
4. ... is not a tautology.
   a. $\exists x \forall x F \longrightarrow \forall x F$
   b. $\forall x(F \wedge G) \longrightarrow (\forall x F \wedge \forall x G)$
   c. $(\exists x F \wedge \exists x G) \longrightarrow \exists x(F \wedge G)$
   d. $F[x/a] \longrightarrow \exists x F$
5. ... is a mgu of $\{P(f(x,y),y), P(z,h(x)), P(f(g(u),y),h(v))\}$
   a. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u),h(g(u)), v/u]$
   b. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u),y), v/g(u)]$
   c. $\sigma = [x/g(u), y/h(x), z/f(g(u),h(g(u))), v/u]$
   d. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u),h(g(u))), v/g(u)]$
6. The substitution ... is a unifier of $S = \{P(x,f(x)), P(g(y),f(z))\}$.
   a. $[x/g(a), y/a, z/g(a)]$
   b. $[x/g(a), y/a, z/x]$
   c. $[x/g(y), z/x]$
   d. $[x/g(a), y/a, z/a]$
7. Let s be a substitution such that $s \diamond s = []$. Then ...
   a. s is a relabeling.
   b. $s = []$
   c. $x/y \in s$ if and only if $y/x \in s$.

8. …is a rename (relabeling).
   a. [x/a, y/x]
   b. $[x/y, y/x]$
   c. $[x/y, y/z, z/y]$
9. Let S be a finite set of FOL clauses. Then, $Res^*[S]$ …
   a. is finite.
   b. is infinite.
   c. is countable.
10. …does not preserve the relation $\equiv_s$.
   a. $\exists x$
   b. $\vee$
   c. $\neg$

Question 7. (15 points)

Prove by first order resolution that the set $S = \{\{P(f(x), y), P(y, f(z)), Q(x, y, f(x))\}, \{\neg P(x, y), Q(z, x, y)\}, \{\neg R(g(x), f(y)), \neg R(g(z), u), \neg Q(x, u, f(z))\}, \{R(x, y), \neg Q(z, y, y)\}\}$ is unsatisfiable. For each resolution step specify the relabelings, the unification set and the mgu. Do the minimal number of unifications. Draw your tree on a blank sheet of paper.

Question 8. (16 points)

We create the data base shown below.

```
arc(b,c).
arc(c,a).
arc(a,b).
arc(a,d).
path1(X,X).
path1(X,Y) :- path1(X,Z), arc(Z,Y).
path2(X,X).
path2(X,Y) :- arc(X,Z), path2(Z,Y).
```

What will be printed out by the queries below?
Write yes, no, or out of local stack next to the query.

?- path1(a,d).

?- path2(a,d).

?- path1(d,a).

?- path2(d,a).

Question 9. (10 points)

Let σ be a mgu of $S = \{A_1, \ldots A_n, B_1, \ldots B_m\}$. Prove that $T_1 = \{A_1, \ldots A_n\}$ and $T_2 = \{B_1, \ldots B_m\}$ have mgu's.

Write your answer on a blank sheet of paper.

Question 10 (20 points)

Let $S = \{A_1, \ldots, A_n\}$ be a unifiable set of atoms, Var be the set of variables that occur in S, and σ be a mgu algorithm produced by the mgu algorithm from the book. Prove that $\sigma = [x_1/t_1, \ldots, x_n/t_n]$ where $x_1, \ldots, x_n \in Var$ and $t_1, \ldots t_n$ have variables in $Var - \{x_1, \ldots, x_n\}$.

Write your answer on a blank sheet of paper.