COT 3420
Sections U2
SPRING 2007

FINAL EXAM

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 10 questions on the test, for a total of 176 points.
3. If you took the induction retest, I will add 9 points to the exam score.
4. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 2 hours and a half to complete the exam.
7. Mark the answers to questions 2,3,4,5,6, and 8 on the exam paper. Write the answers to the other questions on the blank sheets.
8. No talking to each other during the test!
9. Write your name and panther id below.

NAME: _
Panther ID:_ _

QUESTIONS

Question 1. (25 points)

Prove the Subsumption Theorem for FOL. Let C_1 and C_2 be two clauses such that $C_1 \subseteq C_2$. Let $\overline{C_1}$ and $\overline{C_2}$ be the universal closures of C_1 and C_2 in this order. Prove that $\overline{C_1} \wedge \overline{C_2} \equiv \overline{C_1}$.

Write your proof on a blank sheet of paper.

Question 2. (20 points)

Write a prolog predicate count(List,Item,Sum) which is satisfied if List has Sum occurrences of Item. For example, count([a, 3, a, 6, a, 3, a], a, Total) is satisfied if Total = 4.

Write your answer below.

Question 3. (20 points)

Write $E[F, 2]$ for $F = \forall x \forall y \forall z[(P(x, f(x)) \vee P(f(y), a)) \wedge (\neg P(y, f(z)) \vee \neg P(a, z))]$. You can write F^M for $[(P(x, f(x)) \vee P(f(y), a)) \wedge (\neg P(y, f(z)) \vee \neg P(a, z))]$.

Write your answer below.

Question 4. (10 points)

Write all resolvents of the clauses $C_1 = \{P(x, g(y)), P(f(z), z), Q(x, y, z)\}$ and $C_2 = \{\neg P(f(x), y), Q(f(y), z, x)\}$. Don't write the relabelings, the unification set, and the mgu, just display the resolvents. Write your answer below.

Question 5. (10 points)

Find out if $S = \{P(x, f(y), g(x, y)), P(h(y), z, g(h(u), f(v))), P(h(u), f(f(v)), w)\}$ is unifiable. If so, display an mgu; otherwise write that S is not unifiable. Write your answer below.

Question 6. (30 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. $[x/z, z/x, v/u, w/v] \diamond [x/y, y/z, z/x, u/v] = \ldots$
   a. $[x/y, y/x, w/v]$
   b. $[z/y, w/v, y/z, u/v]$
   c. $[z/y, w/v, y/z]$
   d. $[x/y, y/x, v/u, w/v]$
2. $[v/b] \diamond [y/h(z)] \diamond [u/g(y, v)] \diamond [x/f(y, z)] = \ldots$
   a. $[x/f(h(z), z), u/g(h(z), v), y/h(z), v/b]$
   b. $[x/f(h(z), z), u/g(y, n), y/h(z), v/b]$
   c. $[x/f(h(z), z), u/g(h(z), b), y/h(z), v/b]$
   d. $[x/f(h(z), z), u/g(h(z), b), y/h(b), v/b]$
3. $\ldots$ is unifiable.
   a. $\{P(x, y), P(f(x), z)\}$
   b. $\{P(x, f(y)), P(y, a)\}$
   c. $\{P(y, f(z)), P(f(x), y)\}$.
   d. $\{P(f(x), x), P(y, y)\}$.
4. $\ldots$ is not a tautology.
   a. $\exists x P(x) \longrightarrow P(x)$
   b. $P(x, y) \longrightarrow \exists x \exists y P(x, y)$
   c. $\forall x P(x) \longrightarrow \exists x P(x)$
   d. $\forall x \forall y P(x, y) \longrightarrow \forall x P(x, x)$
5. $\ldots$ is an mgu of $\{P(f(x, y), y), P(z, h(x)), P(f(g(u), y), h(v))\}$
   a. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u), h(g(u)), v/u]$
   b. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u), y), v/g(u)]$
   c. $\sigma = [x/g(u), y/h(x), z/f(g(u), h(g(u))), v/u]$
   d. $\sigma = [x/g(u), y/h(g(u)), z/f(g(u), h(g(u))), v/g(u)]$
6. The substitution $\ldots$ is a unifier of $S = \{P(x, f(x), y), P(g(y), z, h(u))\}$.
   a. $[x/g(y), y/h(u), z/f(g(h(u)))]$
   b. $[x/g(a), y/h(a), z/f(g(h(a)))]$
   c. $[x/g(h(a)), y/h(a), z/f(g(h(a))), u/a]$
   d. $[x/g(h(a)), y/h(h(a)), z/f(g(h(a)))]$
7. Let s and σ be two substitution such that $s \diamond \sigma = []$. Then $\ldots$
   a. s is a relabeling.
   b. $s = \sigma = []$
   c. $x/y \in s$ if and only if $y/x \in \sigma$.

8. ...is a ground substitution.

a. [x/a, y/x]

b. $[x/y, y/x]$

c. $[x/y, y/z, z/y]$

d. []

9. Let us define the relation $\uparrow$ on the set of atoms as $P \uparrow Q$ if P and Q are unifiable. Then, $\uparrow$ is ...

a. symmetric.

b. antisymmetric.

c. transitive.

10. The lifting lemma tells us that ...

a. we can lift derivation trees from the Herbrand expansions to FOL.

b. the connectives $\neg, \vee, \wedge, \longrightarrow, \longleftrightarrow, \forall x, \forall y$ preserve equivalences.

c. whenever t is free for x in F, $\mathcal{A}_{[x \leftarrow \mathcal{A}[t]]}[F] = \mathcal{A}[F]$.

d. a set of FOL clauses S is unsatisfiable iff it has a finite subset that is unsatisfiable.

Question 7. (15 points)

Prove by first order resolution that the set $S = \{\{P(x, f(x)), P(g(y), z), Q(x, y, z)\}, \{\neg P(x, y), Q(x, z, y)\}, \{R(g(x), y), \neg R(z, f(x)), \neg Q(z, x, f(z))\}, \{R(x, f(y)), \neg Q(x, y, z)\}\}$ is unsatisfiable. For each resolution step specify the relabelings, the unification set and the mgu. Do the minimal number of unifications. Draw your tree on a blank sheet of paper.

Question 8. (16 points)

We create the data base shown below.

```
arc(a,b).
arc(b,b).
arc(b,c).
arc(c,b).
path1(X,X).
path1(X,Y) :- path1(X,Z), arc(Z,Y).
path2(X,X).
path2(X,Y) :- arc(X,Z), path2(Z,Y).
```

What will be printed out by the queries below?

Write yes, no, or out of local stack next to the query.

?- path1(a,c).

```
?- path2(a,c).

?- path1(c,a).

?- path2(c,a).
```

Question 9. (15 points)

Prove the Compactness Theorem for FOL: A set of formulas F is unsatisfiable, iff it has a finite subset that is unsatisfiable. You may use any theorem in the book except this one. Write your answer on a blank sheet of paper.

Question 10 (15 points)

We can define binary resolution for FOL by unifying only one literal from C_1 with one literal from C_2. Prove that the binary resolution is incomplete by exhibiting a set of clauses that yields the box when we apply the full resolution, but does not produce the box when we do the binary resolution.

Write your answer on a blank sheet of paper.