COT 3420
SUMMER A 2003
Section 1

EXAM # 2

INSTRUCTIONS

1. The test is CLOSED book, CLOSED notebook. You cannot use the practice test either.

2. There are 5 questions on the test, for a total of 95 points.

3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.

4. For the proof/disproof question write proof or disproof and then go on to solve the problem.

5. If you do not understand the meaning of a question ask me during the test.

6. You have 1 hour to work on the test.

7. Write all the answers on the exam paper.

8. Write your name below.

NAME: _____

QUESTIONS

Question 1.(20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. If F is satisfiable, then $\neg F$ is ...
   a. unsatisfiable.
   b. satisfiable.
   c. satisfiable or unsatisfiable, depending on F.
2. If F is satisfiable, then $F \vee G$ is ...
   a. unsatisfiable.
   b. satisfiable.

c. satisfiable or unsatisfiable, depending on F and G.

3. If $F \wedge G$ is satisfiable, then F is ...
   a. unsatisfiable.
   b. satisfiable.
   c. satisfiable or unsatisfiable, depending on F.
4. The formula $F \longrightarrow F$ ...
   a. is a tautology.
   b. is satisfiable.
   c. can be unsatisfiable.
5. If $F \longleftrightarrow G$ and F are satisfiable, then G ...
   a. is satisfiable.
   b. is unsatisfiable.
   c. can be satisfiable or unsatisfiable.
6. If F is satisfiable, then it has ...
   a. a finite number of models.
   b. countably many models.
   c. uncountably many models.
7. If $F \wedge G$ is a tautology then F ...
   a. is a tautology.
   b. is satisfiable.
   c. can be unsatisfiable.
8. If $F \models G$, $F \models H$, and $G, H \models I$ then ...
   a. $F \models I$.
   b. $I \models F$.
   c. $I \models G$.
9. $F, \neg F \models G$ is ...
   a. always true.
   b. always false.
   c. sometimes true and sometimes false.
10. $F, F \longrightarrow G \models G$ is ...
    a. always true.
    b. always false.
    c. sometimes true and sometimes false.

Question 2. (20 points)

Prove by structural induction that the set of connectives $S = \{\neg F \wedge G, \mathbf{T}\}$ is adequate. The formula $\mathbf{T}$ is a tautology.

Write your proof below and on the opposite page.

Question 3. (20 points)

Prove or disprove. If $F \longrightarrow G$ and $H \longrightarrow I$ are satisfiable, and F and H have no atoms in common, then $(F \vee H) \longrightarrow (G \vee I)$ is satisfiable. Write your proof or counter-example below. If you need more space, use the opposite page.

Question 4. (20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. The equivalence $F \wedge G \equiv G \wedge F$ is called ...
   a. associativity.
   b. commutativity.
   c. idempotency.
2. The equivalence relation $\equiv$ is defined on ...
   a. the set of formulas.
   b. the set of truth values.
   c. the set of truth assignments.
3. The equivalence $F \equiv \neg F$ is ...
   a. always true.
   b. always false.
   c. sometimes true and sometimes false.
4. If $F \equiv G$ and $H \equiv I$, then ...
   a. $F \wedge H \equiv F$.
   b. $F \wedge G \equiv H$.
   c. $F \wedge G \equiv G$.
5. If $F \equiv G$ then ...
   a. they have the same models.
   b. they have some models in commun.
   c. they can have disjoint sets of models.
6. If F and G have a common CNF then ...
   a. $F \equiv G$.
   b. $F \not\equiv G$.
   c. sometimes $F \equiv G$ and sometimes $F \not\equiv G$.
7. A formula F has ... CNF's.
   a. finitely many
   b. countably many
   c. uncountably many
8. A clause is ...
   a. a disjunction of literals.
   b. a conjunction of literals.
   c. a formula that contains only atoms and $\longrightarrow$'s.
9. The Substitution Theorem says that ...

a. a formula doesn't change its meaning when we replace a subformula by an equivalent subformula.

b. the truth value of a formula is determined by the truth values of its atoms.

c. every formula is equivalent to a conjunction of clauses.

10. If S is a set of formulas, $Con(S)$ is ...

a. the set of models of S.

b. the set of consequences of S.

c. the set of CNF's of S.

Question 5. (15 points)

Find a CNF for $F = \neg((A \vee \neg B) \longleftrightarrow \neg(B \wedge C))$.

Show your work on the opposite page.