COT 3420
SUMMER A 2003
Section 2

EXAM # 2

INSTRUCTIONS

1. The test is CLOSED book, CLOSED notebook. You cannot use the practice test either.

2. There are 5 questions on the test, for a total of 95 points.

3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.

4. For the proof/disproof question write proof or disproof and then go on to solve the problem.

5. If you do not understand the meaning of a question ask me during the test.

6. You have 1 hour to work on the test.

7. Write all the answers on the exam paper.

8. Write your name below.

NAME: _____

QUESTIONS

Question 1. (20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. If $F \vee G$ is satisfiable, then F is ...
   a. unsatisfiable.
   b. satisfiable.
   c. satisfiable or unsatisfiable, depending on F and G.
2. If G is a tautology then $F \longrightarrow G$ is ...
   a. a tautology.
   b. satisfiable.

c. satisfiable or unsatisfiable, depending on F.

3. If $F \longrightarrow G$ is satisfiable, then $\neg G \longrightarrow \neg F$ is ...
   a. unsatisfiable.
   b. satisfiable.
   c. satisfiable or unsatisfiable, depending on F and G.
4. The formula $(F \wedge G) \longrightarrow F$ ...
   a. is a tautology.
   b. is satisfiable.
   c. can be unsatisfiable.
5. If both $F \longrightarrow G$ and $F \longrightarrow \neg G$ are satisfiable, then F ...
   a. is satisfiable.
   b. is unsatisfiable.
   c. can be satisfiable or unsatisfiable.
6. If $\mathcal{A}$ is a model of F, then ...
   a. $\mathcal{A}$ is a model of $F \wedge G$.
   b. $\mathcal{A}$ is a model of $F \longrightarrow G$.
   c. $\mathcal{A}$ is a model of $F \vee G$.
7. If $F \longrightarrow G$ is a tautology then ...
   a. G must be a tautology.
   b. F must be unsatisfiable.
   c. F can be any formula.
8. If $F \models G$ and $F \models \neg G$ then ...
   a. $F \models H$.
   b. $F \not\models H$.
   c. sometimes $F \models H$ and sometimes $F \not\models H$.
9. If $\neg F \vee G$ is satisfiable, then ... is satisfiable.
   a. $\neg(F \wedge G)$
   b. $\neg(\neg F \wedge G)$
   c. $\neg(F \wedge \neg G)$
10. If F and G have no common models then ...
    a. $F \models \neg G$.
    b. at least one of F, G must be unsatisfiable.
    c. $\neg G \models \neg F$.

Question 2. (20 points)

Prove by structural induction that the set of connectives $S = \{\neg F \wedge \neg G\}$ is adequate.

Write your proof below and on the opposite page.

Question 3. (20 points)

Prove or disprove. If $F \longrightarrow G$ and $H \longrightarrow I$ are satisfiable, then $(F \vee H) \longrightarrow (G \vee I)$ is satisfiable. Write your proof or counter-example below. If you need more space, use the opposite page.

Question 4. (20 points)

For each of the following relations select the string that provides the most accurate description. There is no penalty for wrong guessing, but choose only one answer.

1. The equivalence $F \wedge F \equiv F$ is called ...
   a. associativity.
   b. commutativity.
   c. idempotency.
2. The operation $\boxed{\wedge}$ is defined on ...
   a. the set of formulas.
   b. the set of truth values.
   c. the set of truth assignments.
3. The equivalence $(F \longrightarrow G) \equiv (\neg F \vee G)$ is ...
   a. always true.
   b. always false.
   c. sometimes true and sometimes false.
4. The index of $\equiv$ is
   a. finite.
   b. countably infinite.
   c. uncountably infinite.
5. If $F \models G$ then ...
   a. every model of F is a model of G.
   b. every model of G is a model of F.
   c. they can have disjoint sets of models.
6. If F and G have no common CNF then ...
   a. $F \equiv G$.
   b. $F \not\equiv G$.
   c. sometimes $F \equiv G$ and sometimes $F \not\equiv G$.
7. Let C_1 and C_2 be two clauses. If $C_1 \subseteq C_2$, then ...
   a. $C_1 \wedge C_2 \equiv C_1$.
   b. $C_1 \wedge C_2 \equiv C_2$.
   c. $C_1 \wedge C_2$ does not have to be equivalent to neither C_1 nor C_2.
8. The number of non-equivalent clauses that can be formed with n atoms is approximated by ...
   a. 2^n.
   b. 3^n.
   c. 4^n.

9. The CNF Theorem says that ...

a. a formula doesn't change its meaning when we replace a subformula by an equivalent subformula.

b. the truth value of a formula is determined by the truth values of its atoms.

c. every formula is equivalent to a conjunction of clauses.

10. Let $S \subseteq T$ be two sets of formulas. Then, ...

a. if T is unsatisfiable, so is S.

b. if S is satisfiable, so is T.

c. if S is unsatisfiable, so is T.

Question 5. (15 points)

Find a CNF for $F = ((A \vee \neg B) \wedge C) \longleftrightarrow \neg(B \wedge D)$.

Show your work on the opposite page.