COT 3420
SUMMER A 2003
Section 2

EXAM # 2 ANSWERS

Question 1.(20 points)
1. c 2. a 3. b 4. a 5. c 6. c 7. c 8. a 9. c 10. a
Grading Criteria: 2 points for each correct answer.

Question 2. (20 points)
Proof: We write $F \downarrow G$ for $\neg F \wedge \neg G$. So, the set of connectives is $S = \{\downarrow\}$. The S-formulas are defined below.
1. the atoms are S-formulas,
2. if F and G are S-formulas, so is $F \downarrow G$.

We prove, by structural induction, that every formula F has an S-equivalent formula.
Case 1: F is an atom. Then F is an S formula.
Case 2: $F = \neg G$. By induction hypothesis there is an S-formula G_1 such that $G \equiv G_1$. Then
$F = \neg G$
$\equiv \neg G_1$ by induction hypothesis
$\equiv \neg G_1 \wedge \neg G_1$ by idempotency
$= G_1 \downarrow G_1$ by the definition of $\downarrow$
The last formula is an S-formula.
Case 3: $F = G \vee H$. By induction hypothesis there are S-formulas G_1 and H_1 such that $G \equiv G_1$ and $H \equiv H_1$.
$F = G \vee H$
$\equiv G_1 \vee H_1$ by induction hypothesis
$\equiv \neg\neg(G_1 \vee H_1)$ double negation introduction
$\equiv \neg(\neg G_1 \wedge \neg H_1)$ De Morgan's Law
$= \neg(G_1 \downarrow H_1)$ by the definition of $\downarrow$
$\equiv (G_1 \downarrow H_1) \downarrow (G_1 \downarrow H_1)$ by Case 2
The last formula is an S-formula.
Case 4: $F = G \wedge H$. By induction hypothesis there are S-formulas G_1 and H_1 such that $G \equiv G_1$ and $H \equiv H_1$.

$F = G \wedge H$
$\equiv G_1 \wedge H_1$ by induction hypothesis
$\equiv \neg\neg G_1 \wedge \neg\neg H_1$ by double negation introduction
$= \neg G_1 \downarrow \neg H_1$ by the definition of $\downarrow$
$\equiv (G_1 \downarrow G_1) \downarrow (H_1 \downarrow H_1)$ by Case 2
The last formula is an S-formula.

Case 5: $F = G \longrightarrow H$. By induction hypothesis there are S-formulas G_1 and H_1 such that $G \equiv G_1$ and $H \equiv H_1$.
$F = G \longrightarrow H$
$\equiv G_1 \longrightarrow H_1$ by induction hypothesis
$\equiv \neg G_1 \vee H_1$ $\longrightarrow$-elimination
$\equiv (\neg G_1 \downarrow H_1) \downarrow (\neg G_1 \downarrow H_1)$ by Case 3
$\equiv ((G_1 \downarrow G_1) \downarrow H_1) \downarrow ((G_1 \downarrow G_1) \downarrow H_1)$ by Case 2
The last formula is S.

Case 6: $F = G \longleftrightarrow H$. By induction hypothesis there are S-formulas G_1 and H_1 such that $G \equiv G_1$ and $H \equiv H_1$.
$F = G \longleftrightarrow H$
$\equiv G_1 \longleftrightarrow H_1$ by induction hypothesis
$\equiv (G_1 \longrightarrow H_1) \wedge (H_1 \longrightarrow G_1)$ by $\longleftrightarrow$-elim
$\equiv (\neg G_1 \vee H_1) \wedge (\neg H_1 \vee G_1)$ $\longrightarrow$-elimination
$\equiv (\neg G_1 \vee \neg\neg H_1) \wedge (\neg H_1 \vee \neg\neg G_1)$ double negation introduction
$\equiv \neg(G_1 \wedge \neg H_1) \wedge \neg(H_1 \wedge \neg G_1)$ De Morgan's law
$= (G_1 \wedge \neg H_1) \downarrow (H_1 \wedge \neg G_1)$ by the definition of $\downarrow$
$\equiv (\neg\neg G_1 \wedge \neg H_1) \downarrow (\neg\neg H_1 \wedge \neg G_1)$ double negation introduction
$= (\neg G_1 \downarrow H_1) \downarrow (\neg H_1 \downarrow G_1)$ by the definition of $\downarrow$
$\equiv ((G_1 \downarrow G_1) \downarrow H_1) \downarrow ((H_1 \downarrow H_1) \downarrow G_1)$ by Case 2
The last formula is S.

Grading Criteria: 1. Listing the 6 cases : 4 points.
2. Case 1: 1 point.
3. Cases 2,3,4,5,6: 3 points each.

For cases 2, 3,4,5,6 the IH and reasons are worth 0.5 points each. You got points off if the S-formulas are too long.

Question 3. (20 points)
Counter-example: Let $F = P_1$, $G = P_1 \wedge \neg P_1$, $H = \neg P_1$, $I = P_1 \wedge \neg P_1$. Below are the truth tables of $F \longrightarrow G$, $H \longrightarrow I$, $(F \vee H) \longrightarrow (G \vee I)$.

P_1	F	G	H	I	$F \longrightarrow G$	$H \longrightarrow I$	$F \vee H$	$G \vee I$	$(F \vee H) \longrightarrow (G \vee I)$
0	0	0	1	0	1	0	1	0	0
1	1	0	0	0	0	1	1	0	0

The formulas $F \longrightarrow G$ and $H \longrightarrow I$ are satisfiable, but $(F \vee H) \longrightarrow (G \vee I)$ is not.

Grading Criteria: 1. If you did not right **Proof** or **Disproof** before you starting your work, you cannot get more than 2 points.

1. If you make the wrong choice: 3 points.

2. If you make the correct choice : 6 points. The correct choice for F,G,H,I is worth 2 points each. The truth table is worth 6 points.

Question 4. (20 points)

1. c 2. b 3. a 4. b 5. a 6. b 7. a 8. b 9. c 10. c

Grading Criteria: 2 points for each correct answer.

Question 5. (15 points)

$F = ((A \vee \neg B) \wedge C) \longleftrightarrow \neg(B \wedge D)$

$\equiv [((A \vee \neg B) \wedge C) \longrightarrow \neg(B \wedge D)] \wedge [\neg(B \wedge D) \longrightarrow ((A \vee \neg B) \wedge C)]$ $\longleftrightarrow$-elimination

$\equiv [\neg((A \vee \neg B) \wedge C) \vee \neg(B \wedge D)] \wedge [\neg\neg(B \wedge D) \vee ((A \vee \neg B) \wedge C)]$ $\longrightarrow$-elimination

$\equiv [(\neg(A \vee \neg B) \vee \neg C \vee \neg B \vee \neg D] \wedge [(B \wedge D) \vee ((A \vee \neg B) \wedge C)]$ De Morgan's law twice, $\neg\neg$ elimination

$\equiv [(\neg A \wedge \neg\neg B) \vee \neg C \vee \neg B \vee \neg D] \wedge [(B \wedge D) \vee ((A \vee \neg B) \wedge C)]$ De Morgan's law

$\equiv [(\neg A \wedge B) \vee \neg C \vee \neg B \vee \neg D] \wedge [(B \wedge D) \vee ((A \vee \neg B) \wedge C)]$ double negation elimination,

$\equiv (\neg A \vee \neg C \vee \neg B \vee \neg D) \wedge (B \vee \neg C \vee \neg B \vee \neg D) \wedge [(B \wedge D) \vee (A \vee \neg B)] \wedge [(B \wedge D) \vee C]$ by distributivity

$\equiv (\neg A \vee \neg C \vee \neg B \vee \neg D) \wedge (B \vee A \vee \neg B) \wedge (D \vee A \vee \neg B) \wedge (B \vee C) \wedge (D \vee C)$ tautology removal, distributivity twice

$\equiv (\neg A \vee \neg B \vee \neg C \vee \neg D) \wedge (A \vee \neg B \vee D) \wedge (B \vee C) \wedge (C \vee D)$ tautology removal, arranging the literals

Grading: You got credit up the line where you made the first error. For each correct line you got 1.5 points (up to 9), and 1.5 points for stating the reasons.