COT 3420
SUMMER A 2003
SECTION 1

EXAM # 3

INSTRUCTIONS

1. The test is CLOSED book, CLOSED notebook. You cannot use the practice test either.
2. There are 6 questions on the test, for a total of 105 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to work on the test.
5. Write all the answers on the exam paper.
6. Write your name below.

NAME: _

QUESTIONS

Question 1.(20 points)

Select the string that provides the best completion. There is no penalty for wrong guessing, but choose only one answer.

1. Let $C_1 = \{A, B, C\}$ and $C_2 = \{A, B\}$. Then ...
   a. C_1 subsumes C_2.
   b. C_2 subsumes C_1.
   c. neither clause subsumes the other.
2. Let R be a resolvent of C_1 and C_2. If C_1 is a tautology, then ...
   a. R is a tautology.
   b. R is not a tautology.
   c. sometimes R is a tautology and sometimes it is not.
3. The letter i in P_i^n is ...
   a. the arity of the predicate.

b. the index of the predicate.
c. the position of the symbol in the list of all predicate symbols.

4. If $Res^3[S] = Res^4[S]$ then ...
a. S is finite.
b. $Res^*[S]$ is finite.
c. $Res^*[S] = Res^3[S]$.

5. The equality $Res^*[S] = Res[Res^*[S]]$ is ...
a. always true.
b. never true
c. sometimes true and sometimes false.

6. ... is a term.
a. P_1^0
b. f_0^1
c. f_1^0

7. The Resolution Theorem says that ...
a. a set of clauses is unsatisfiable iff $\square$ is one of its resolvents.
b. if $Res^n[S] = Res^{n+1}[S]$ then $Res^*[S] = Res^n[S]$.
c. a set of formulas is unsatisfiable iff it has a finite unsatisfiable subset.

8. ... is an atomic formula.
a. P_0^1
b. $E(x_1)$
c. $\neg P_0^0$
d. P_1^0

9. Let S be an infinite set of non-equivalent clauses. If S is unsatisfiable, then S has ... many unsatisfiable subsets.
a. finitely
b. countably
c. uncountably

10. In a N-resolution tree every resolvent must ...
a. have a negative parent.
b. have a positive parent.
c. have a unit clause as a parent.
d. not have as parent a tautology clause.

Question 2. (15 points)

Let S be a set of clauses and $Tree[S, n]$ be the set of clauses that have S-resolution trees of height less than or equal to n. Prove that for all $n \in N$, $Tree[S, n] = Res^n[S]$.

Write your answer below and on the opposite page.

Question 3. (15 points)

Let F^* be the string obtained from F by deleting all substrings of the form $\forall x$ and $\exists x$, where x is a variable. Prove, by structural induction on F, that F^* is a formula.

Write your answer below and on the opposite page.

Question 4. (20 points)

Construct a resolution tree of $\Box$ from the set $S = \{\{A, B, C\}, \{A, \neg B, C\}, \{\neg A, D, E\}, \{\neg A, D, \neg E\}, \{\neg A, \neg D\}, \{\neg C, F\}, \{\neg C, \neg F\}\}$.

Draw your tree on the opposite page.

Question 5. (20 points)

Let S be a set of clauses and L a literal that occurs in some clauses of S. The clauses of S fall into 4 categories:

Category 1: The clauses that contain both L and its complement $\bar{L}$.

Category 2: The clauses that contain L, but not $\bar{L}$.

Category 3: The clauses that contain $\bar{L}$, but not L.

Category 4: The clauses that contain neither L nor $\bar{L}$.

We define the sets of clauses $S_{L=0}$ and $S_{L=1}$ as follows:

$C \in S_{L=0}$ if C is a clause in the fourth category, or is obtained from a clause in the second category by removing all occurrences of L, and

$C \in S_{L=1}$ if C is a clause in the fourth category, or is obtained from a clause in the third category by removing all occurrences of $\bar{L}$.

For example, let $S = \{\{A, B, C\}, \{A, C, \neg C\}, \{\neg A, B, C\}, \{B, \neg C\}, \{\neg B\}\}$ and let $L = C$.

The 4 categories are

Category 1: $\{\{A, C, \neg C\}\}$

Category 2: $\{\{A, B, C\}, \{\neg A, B, C\}\}$

Category 3: $\{\{B, \neg C\}\}$

Category 4: $\{\{\neg B\}\}$

We get $S_{C=0}$ by removing all C's from the clauses of the second category and uniting them with the 4th category. We obtain $S_{C=0} = \{\{A, B\}, \{\neg A, B\}, \{\neg B\}\}$.

We get $S_{C=1}$ by removing all $\neg C$'s from the 3rd category and uniting them with the 4th category. We obtain $S_{C=1} = \{\{B\}, \{\neg B\}\}$.

Prove the lemma below.

Lemma : Let S be a set of clauses and L a literal that occur in some clauses of S. Then S is unsatisfiable iff both $S_{L=0}$ and $S_{L=1}$ are unsatisfiable.

Write your proof on the opposite page.

Question 6. (15 points)

The structure $\mathcal{A}$ has universe $D = \{3, 4, 5\}$, $x^{\mathcal{A}} = 4$, $y^{\mathcal{A}} = 5$, and $a^{\mathcal{A}} = 3$. The interpretations of the unary function f, binary function g, and binary predicate P are shown in Figure 1.

x	$f^{\mathcal{A}}[x]$
3	4
4	5
5	3

$x \backslash y$	3	4	5
3	4	3	5
4	5	4	3
5	3	4	4

$g^{\mathcal{A}}[x, y]$

$x \backslash y$	3	4	5
3	0	1	0
4	0	0	0
5	1	1	1

$P^{\mathcal{A}}[x, y]$

Figure 1: Tables for Question 6

Find the interpretation of the terms and formulas below. Write the answer next to the equal sign. Do not show your work.

1. $\mathcal{A}[f(x)] =$

2. $\mathcal{A}[f(g(a, y))] =$

3. $\mathcal{A}[P(x, y)] =$

4. $\mathcal{A}[\forall x P(y, x)] =$

5. $\mathcal{A}[\exists y P(x, y)] =$