COT 3420
SUMMER A 2003
SECTION 2

EXAM # 3

INSTRUCTIONS

1. The test is CLOSED book, CLOSED notebook. You cannot use the practice test either.
2. There are 6 questions on the test, for a total of 105 points.
3. For the multiple choice questions, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to work on the test.
5. Write all the answers on the exam paper.
6. Write your name below.

NAME: _

QUESTIONS

Question 1.(20 points)

Select the string that provides the best completion. There is no penalty for wrong guessing, but choose only one answer.

1. Let R and S be two sets of clauses. If $R \subseteq S$, then $Res^*[S] \subseteq Res^*[T]$ is ...

a. always true.
b. never true.
c. sometimes true ans sometimes false.

2. Let C be a clause. If $C \neq \Box$, then ...

a. C is satisfiable.
b. C is tautology.
c. C is unsatisfiable.

3. The letter i in P_i^n is ...

a. the arity of the predicate.
b. the index of the predicate.
c. the position of the symbol in the list of all predicate symbols.

4. If $\Box$ is a resolvent of C_1 and C_2, then ...
a. both must be unit clauses.
b. at least one must be a unit clause.
c. both clauses can have more than one element.

5. The equality $Res^*[Res^*[S]] = Res^*[S]$ is ...
a. always true.
b. never true
c. sometimes true and sometimes false.

6. ...is a term.
a. P_0^0
b. f_0^1
c. f_1^0

7. The Agreement Theorem for FOL says that ...
a. a set of clauses is unsatisfiable iff $\Box$ is one of its resolvents.
b. the truth value of a formula is determined by the universe of the structure and the interpretation of the functions, predicates, and free variables of the formula.
c. a set of formulas is unsatisfiable iff it has a finite unsatisfiable subset.

8. ...is an atomic formula.
a. P_0^1
b. $E(x_1)$
c. $\neg P_0^0$
d. P_1^0

9. Let S be a set of clauses such that $S = Res^*[S]$, and let C_1, C_2, be two clauses in S. Then, ...
a. C_1 and C_2 are not unifiable.
b. their resolvents are in S.
c. if they are unifiable, their resolvents are in S.

10. Let $\mathcal{A}$ and $\mathcal{B}$ be two structures with the same universe. If $\mathcal{A}$ and $\mathcal{B}$ agree on $\forall x(F \vee G)$, then they agree on ...
a. F.
b. G.
c. $F \vee G$.
d. $\exists x(F \vee G)$.

Question 2. (15 points)

Prove that $Res[Res^*[S]] = Res^*[S]$.

Write your answer below and on the opposite page.

Question 3. (20 points)

Let $n[fun, t]$ be the number of function symbols of arity greater than 0 that occur in the term t. Prove, by structural induction on t, that for every prefix S of t, $n[fun, S] \geq n[(, S] \geq n[), S]$.

Write your answer below and on the opposite page.

Question 4. (20 points)

Construct a resolution tree of $\square$ from the set $S = \{\{A, B, \neg C\}, \{\neg A, B, \neg C\}, \{C, D, E\}, \{C, \neg D, E\}, \{C, \neg E\}, \{\neg B, E, F\}, \{\neg B, E, \neg F\}, \{\neg B, \neg E\}\}$.

Draw your tree on the opposite page.

Question 5. (15 points)

Let $S = \{F_0, \ldots, F_n, \ldots\}$ be an infinite set of formulas. Prove that S is satisfiable iff $\bigwedge_{i=0}^{n} F_i$ is satisfiable for infinitely many n's.

Write your proof below and on the back of this page.

x	$f^{\mathcal{A}}[x]$
3	5
4	4
5	3

$x \backslash y$	3	4	5
3	3	4	5
4	4	5	3
5	5	3	4

$g^{\mathcal{A}}[x, y]$

$x \backslash y$	3	4	5
3	1	1	1
4	0	1	0
5	0	0	0

$P^{\mathcal{A}}[x, y]$

Figure 1: Tables for Question 6

Question 6. (15 points)

The structure $\mathcal{A}$ has universe $D = \{3, 4, 5\}$, $x^{\mathcal{A}} = 3$, $y^{\mathcal{A}} = 4$, and $a^{\mathcal{A}} = 5$. The interpretations of the unary function f, binary function g, and binary predicate P are shown in Figure 1.

Find the interpretation of the terms and formulas below. Write the answer next to the equal sign. Do not show your work.

1. $\mathcal{A}[f(x)] =$

2. $\mathcal{A}[f(g(a, y))] =$

3. $\mathcal{A}[P(x, y)] =$

4. $\mathcal{A}[\forall y P(x, y)] =$

5. $\mathcal{A}[\exists x P(a, x)] =$