COT 3420
SUMMER A 2003

PRACTICE EXAM # 4

INSTRUCTIONS

1. The test is OPEN book, OPEN notebook. You can use the practice test.
2. There are 7 questions on the test, for a total of 70 points. There is also a bonus question worth 15 points.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to work on the test.
5. Write all the answers on the exam paper.
6. Write your name below.

NAME: _

QUESTIONS

Question 1.(5 points)
Skolemize the formula $F = \exists x \forall y \exists z \exists u \forall v \exists w F^M[x, y, z, u, v, w]$.

Question 2. (10 points)
Rectify the formula $F = \forall x(P(x, y) \vee \exists x P(x, z)) \wedge \forall y(\neg P(y, x) \vee \forall z \neg P(z, f(y)))$.

Question 3. (5 points)
Close the formula $F = \forall z \exists v F^M[x, y, z, u, v, w]$.

Question 4. (10 points)
Prove that if x is not free in G, then
$\forall x F \longrightarrow G \equiv \exists x(F \longrightarrow G)$.

Question 5. (15 points)
Let S be the set that contains all atomic formulas, the empty clause, and the operators $\longrightarrow$ and $\exists x$, where x can be any variable. Show, by structural induction, that S is adequate.

Question 6. (15 points)

Construct a derivation tree of $\square$ from $S = \{\{\neg P(x,y), \neg P(y,z), Q(x,f(z))\}, \{P(x,y), Q(x,z)\}, \{\neg Q(a,x), \neg R(x,y)\}, \{R(f(x),a), R(y,x)\}\}$.

Use the minimal number of steps.

Question 7. (10 points)

Write $E(F,2)$ for $F = \forall x \forall y ((P(a,f(x)) \vee P(f(y),x)) \wedge \neg P(b,y))$.

Bonus Question (15 points)

Let C be a clause and s a substitution. We call the clause $s[C]$ a factoring of C. For example, $\{\neg P(x,x)\}$ is a factoring of $\{\neg P(x,y), \neg P(y,z)\}$ because $\{\neg P(x,y), \neg P(y,z)\}[y/x, z/x] = \{\neg P(x,x)\}$. We recall that the binary resolution unifies one literal from clause C_1 with one literal from clause C_2. Prove that the full resolution can be implemented with binary resolution and factoring.