COT 3420
Section U1
Spring 2005

EXAM # 1

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 5 questions on the test, for a total of 121 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 1 hour to complete the exam.
5. Mark the answers to questions 1 and 4 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _

QUESTIONS

Question 1. (36 points)

For each of the following statements select the string that best completes its meaning. There is no penalty for wrong guessing, but choose only one answer.

1. The set of all strings of the object language is ...
   a. finite
   b. countably infinite
   c. uncountably infinite

2. If 0.2.1 is a Dewey address in the tree t, then ... must be an address in the tree.
   a. 1
   b. 0.2.2
   c. 0.1.1

d. 0.1

3. Let $Mod[F] = \{\mathcal{A} | \mathcal{A}[F] = 1\}$, i.e. $Mod[F]$ is the set of all truth assignments that make F true. Then $Mod[F]$ cannot be ...

a. finite.

b. countably infinite.

c. uncountably infinite.

4. If F is a formula, then it ...

a. has more atoms than left parentheses.

b. has more left parentheses than atoms.

c. has as many left parentheses as atoms.

d. can have more left parentheses than atoms, more atoms than left parentheses, or an equal number.

5. The string ... is not a suffix of **Marquito**.

a. λ

b. **Marquito**

c. **Mar**

d. **ito**

6. If $F = (G \vee H)$, $G = XY$, $H = UV$ are formulas, and X, Y, U, V are not empty, then $Y \vee U$ ...

a. is not a formula.

b. has more left parentheses than right parentheses.

c. has more right parentheses than left parentheses.

d. has an equal number of left and right parentheses.

7. $\bigvee_{i=3}^{1} F_i = \ldots$

a. $\square$.

b. **T**.

c. $((F_1 \vee F_2) \vee F_3)$.

d. $(F_1 \vee (F_2 \vee F_3))$.

8. Let F be a formula and S be a non-empty suffix of F. If S is shorter than F, then ...

a. S is not a formula.

b. S is a formula.

c. sometimes S is a formula, and other times not.

9. Let F be a formula of length 10. Then F can have at most ... subformulas.

a. 5

b. 8

c. 9

d. 10

10. The domain of $\mathcal{T}^{ext}$ is ...
   a. $FORM$.
   b. the set of atomic formulas.
   c. $\{0, 1\}$.
11. The domain of $\vee$ is ...
   a. $FORM$.
   b. the set of all atomic formulas.
   c. the set $\{0, 1\} \times \{0, 1\}$.
   d. $FORM \times FORM$.
12. We can form ... finite sets of formulas.
   a. finitely many
   b. countably many
   c. uncountably many

Question 2. (40 points)

Prove that for all formulas F, $n[atom, F] = n[(, F] + 1$. Recall that $n[atom, F]$ is the number of atom occurrences in F.

Question 3. (15 points)

Draw the formula tree of $F = (((\neg P_0 \vee P_1) \wedge \neg(P_2 \longleftrightarrow \neg P_3)) \vee ((P_2 \longleftrightarrow P_3) \wedge (P_4 \vee \neg(P_1 \longrightarrow (P_5 \vee (P_6 \longleftrightarrow \neg P_3))))))$.

Question 4. (15 points)

Write the formula that corresponds to the tree from Figure 1.

Write your answer below.

Question 5. (15 points)

Prove the following proposition:

If S is a non-empty suffix of a formula F and $n[(, S] = n[), S]$, then S is formula.

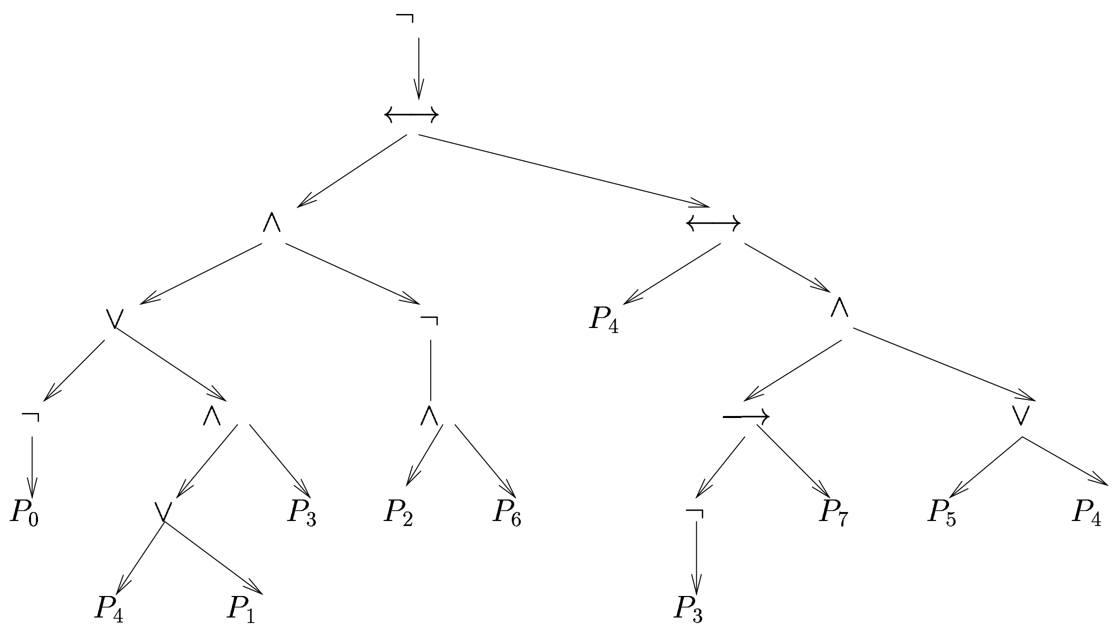

Figure 1: The tree for Question 4