COT 3420
Section U1
Spring 2005

FINAL EXAM

INSTRUCTIONS

1. The test is open book, open notebook.
2. There are 12 questions on the test, for a total of 195 points.
3. For the multiple choice question, there is no penalty for wrong guessing. For proofs, every word counts.
3. If you do not understand the meaning of a question ask me during the test.
4. You have 2 hours and a half to complete the exam.
5. Mark the answers to questions 2,3,4,5,6,7, and 9 on the exam paper. Write the answers to the other questions on the blank sheets.
6. No talking to each other during the test!
7. Write your name below.

NAME: _____

QUESTIONS

Question 1. (20 points)

Let $F = \forall x \forall y F^M$ be a formula with matrix F^M, and f,g be two function symbols of arity 2. Let u and v be two variables that do not occur in F.

Prove that $F \models \forall u \forall v F^M[x/f(u,v), y/g(u,v)]$. Write your proof on a blank sheet of paper.

Question 2. (10 points)

Rectify the formula $F = \forall x\{\forall y[\exists x(P(x,y) \vee \neg Q(z)) \wedge \forall z(\neg P(x,z) \vee Q(y))] \wedge \exists z[\forall x(P(x,z) \vee Q(y)) \wedge \exists z(\neg Q(z) \vee \neg P(z,x))]\}$.

Write your answer below.

Question 3. (5 points)

Close the formula $F = \forall x \exists y F^M[x, y, z, u, v, w]$ where F^M, the matrix of F, has free occurrences of x, y, z, u, v, w.

Write your answer below.

Question 4. (10 points)

Skolemize the formula $F = \exists x \exists y \forall z \forall u \exists v \exists w F^M$, where F^M, the matrix of F, contains the constants a and b and the function symbols f and h.

Write your answer below.

Question 5. (10 points)

Find a prenex form for

$F = \neg\{\forall x[\exists y(P(x, y) \vee \neg Q(x, y)) \wedge \forall z Q(x, z)] \vee \neg \exists u \forall v \exists w(\neg P(u, v) \wedge Q(v, w))\}$.

Write your answer below.

Question 6. (10 points)

Find a mgu for $S = \{P(x, f(x, y)), P(g(y), f(g(z), y)), P(u, f(u, h(v)))\}$.

Write your answer below.

Question 7. (30 points)

The universe of $\mathcal{A}$ is $\{3, 4, 5\}$ and the $\mathcal{A}$ interpretations of a,x, and y are

x	$f^{\mathcal{A}}[x]$
3	3
4	5
5	4

$f^{\mathcal{A}}$

x \ y	3	4	5
3	3	4	5
4	4	5	3
5	5	3	4

$g^{\mathcal{A}}$

x \ y	3	4	5
3	1	0	0
4	0	0	1
5	0	1	0

$P^{\mathcal{A}}$

x	$Q^{\mathcal{A}}[x]$
3	0
4	0
5	1

$Q^{\mathcal{A}}$

Figure 1: Tables for Question 7

$a^{\mathcal{A}} = 4$, $x^{\mathcal{A}} = 3$ and $y^{\mathcal{A}} = 5$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicates $P^{\mathcal{A}}$ and $Q^{\mathcal{A}}$ are displayed in Figure **??**.

Evaluate the terms and the formulas below. Do not show your work, just write the answer to the right of the equal sign.

1. $\mathcal{A}[f(f(y))] =$
2. $\mathcal{A}[g(g(a, x), y)] =$
3. $\mathcal{A}[\neg P(a, x)] =$
4. $\mathcal{A}[\forall x P(x, f(x))] =$
5. $\mathcal{A}[\exists x \forall y P(x, y)] =$
6. $\mathcal{A}[\forall x(Q(x) \vee Q(f(x)))] =$

Question 8. (20 points)

Prove by first order resolution that $\{\{P(x, y), P(y, z), Q(x, z)\}, \{\neg P(x, y), Q(a, x)\}, \{R(x, f(x)), \neg Q(x, a)\}, \{\neg R(a, y), \neg R(x, y), \neg Q(x, x)\}\}$ is unsatisfiable. For each resolution step specify the relabelings, the unification set and the mgu. Do the minimal number of unifications. Draw your tree on a blank sheet of paper.

Question 9. (20 points)

We create the data base shown below.

```
arc(a,b).
arc(b,c).
arc(c,a).
path1(X,X).
path1(X,Y) :- arc(X,Z), path1(Z,Y).
path2(X,X).
```

path2(X,Y):- path2(X,Z), arc(Z,Y).

What will be printed out by the queries below? Write yes, no, or infinite loop next to the query.

?- path1(a,c).

?- path2(a,c).

?- path1(d,a).

?- path2(d,a).

Question 10. (20 points)

The matrix of the formula F is the string obtained from F by deleting all quantifiers Qx. Prove, by structural induction on F, that the matrix of F is a formula. Write your proof on a blank sheet of paper.

Question 11. (20 points)

Prove the consequence below.

$\forall x(F \vee G), \neg F[x/a] \models \exists x G$.

Write your answer on a blank sheet of paper.

Question 12. (20 points)

Assume that y is not free in G. Prove that $\forall y F \longleftrightarrow G \equiv \forall y(F \longleftrightarrow G)$ is not always true.

Write your answer on a blank sheet of paper.