

Lesson 3: Goals and Rules

The topics

1. matching goals and rules
2. the path program
- 3: the path2 program

1. Matching and Rules

In Lesson 2 we learned how prolog handles goals when the clauses are facts. But what happens when we have rules?

At any time, prolog has a stack of goals, G_1, G_2, \dots, G_n that needs to be satisfied. In that lesson we learned that it tries to satisfy G_1 first, by trying to match it with the clauses in the database, from the first down to the last. Let's assume that the clause is the rule below.

$$H : -B_1, \dots, B_m$$

Prolog first relabels the variables of the rule, such that the rule and the stack have no variables in common. So, we get the rule

$$H' : -B'_1, \dots, B'_m.$$

The interpreter then tries to match G_1 and H' . If it succeeds, it puts B'_1, \dots, B'_m on the top of the stack, so, the stack of goals is now $B'_1, \dots, B'_m, G_2, \dots, G_n$. If it fails, prolog tries to match G_1 with the clause that follows the rule.

Let us trace the program prog3, displayed beneath.

```
% define the path based on the arc predicate.  
path(X,X).          % clause 1  
path(X,Y) :- arc(X,Z), path(Z,Y).      % clause 2
```

```
% the arc clauses  
arc(a,b).          % clause 3  
arc(b,c).          % clause 4  
arc(a,d).          % clause 5  
arc(c,e).          % clause 6
```

The path is defined recursively: there is a path from a node to itself (the empty path) and there is a path from X to Y, if there is an arc from X to

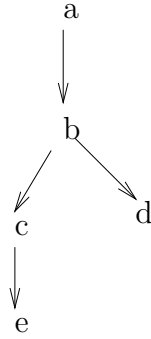


Figure 1: The graph for prog3

some node Z , and a path from Z to Y . The arc clauses describe the graph from Figure 1.

We compile this file and then we ask if there is a path from a to e . The trace of the program is shown in Figure 2

The original goal is $\text{path}(a,e)$. Prolog tries to match it with clause 1 and fails because X cannot be matched with both a and e . So, the interpreter tries to match it with the second clause. This clause is a rule, and there is no need to relabel the clauses of the rule because the goal has no variables. The goal can be matched with the head of the rule and we get the substitution $\{ X = a, Y = b \}$. we apply this substitution to the body and we put the body at the top of the stack of goals, getting $\text{arc}(a,Z), \text{path}(Z,e)$. Prolog tries to satisfy the first goal. It does not match the first 2 clauses because both the first fact and the head of the rule have the functor path . It succeeds with fact 3 and we get the substitution $\{ Z = b \}$. The stack of goals is now $\text{path}(b,e)$. Again, the goal cannot be matched with clause 1, but it succeeds with clause 2, the matching substitution being $\{ X=b, Y=e \}$ and the list of goals is now $\text{arc}(b,Z), \text{path}(c,e)$.

The goal $\text{arc}(b,Z)$ is satisfied by clause 4 with the substitution $\{ Z = c \}$ and the list of goals becomes $\text{path}(c,e)$. This goal is satisfied by clause 2 with the substitution $\{ X=c, Y=e \}$ and the stack of goals is $\text{arc}(c,Z), \text{path}(Z,e)$. The goal $\text{arc}(c,z)$ is satisfied by clause 6, with the substitution $\{ Z=c \}$ leaving us with the goal $\text{path}(e,e)$. The last goal is satisfied by clause 1 and prolog write

true

2. The path predicate

Let's run more queries with the predicate path from prog3. We ask the

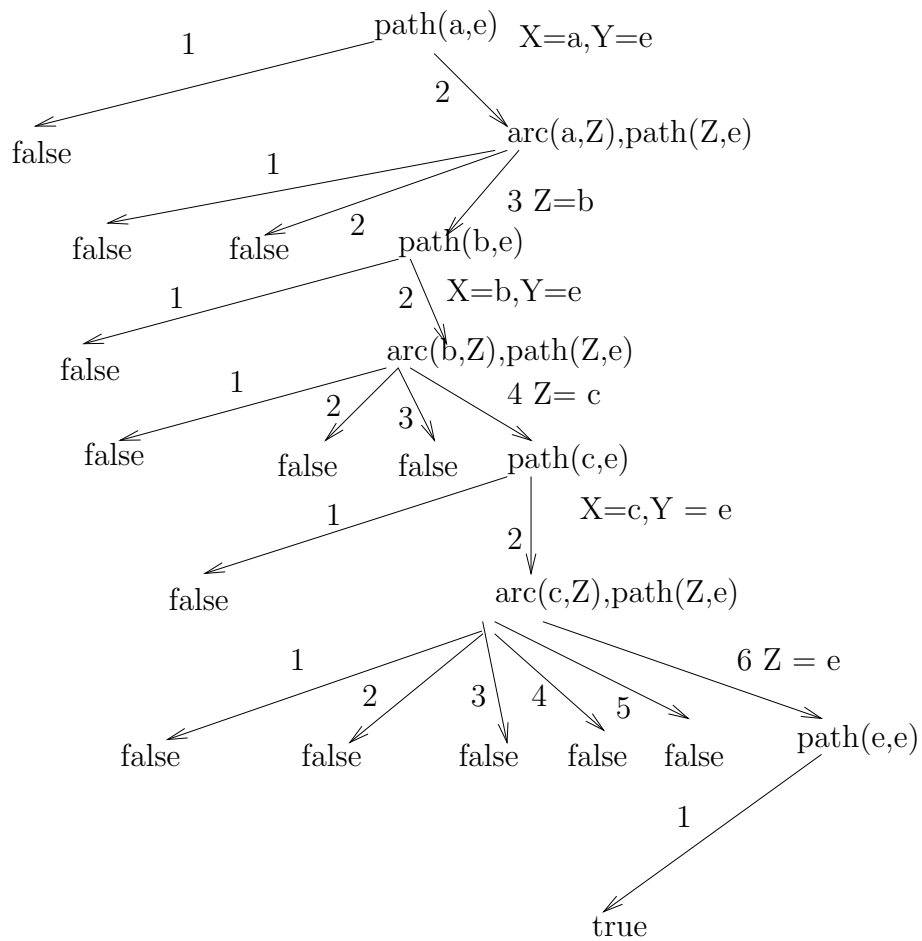


Figure 2: The trace for the query $\text{path}(a,e)$ for prog3

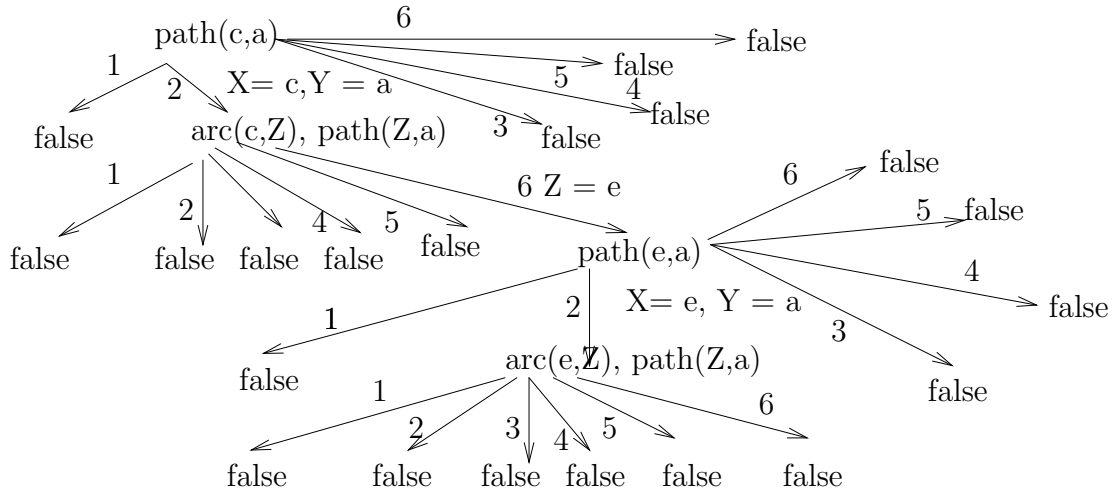


Figure 3: The trace for the query `path(c,a)` for `prog3`

query

?- `path(c,a)`.

The interpreter prints

`false`.

as it should.

Figure 3 traces the execution of this query.

After the goal `arc(e,Z)` fails, the interpreter backtracks to `path(e,a)` and tries to resatisfy it with clauses 3,4,5,6. It fails because the functor of the goal is different from the functor of the clauses. This is shown with the dash arrows. from `path(e,a)`, the interpreter backtracks `path(c,a)` and again tries to resatisfy this goal with the clauses 3,4,5,6. again, it fails for the same reason as `path(e,a)`. So, the top goal fails and the computer prints

`false`.

However, there are problems with the `path` predicate, as illustrated in `prog4.pl`.

```
% define the path based on the arc predicate.
```

```
path(X,X).           % clause 1
```

```
path(X,Y) :- arc(X,Z), path(Z,Y).           % clause 2
```

```
% the arc clauses
```

```
arc(a,b).           % clause 3
```

```
arc(b,c).           % clause 4
```

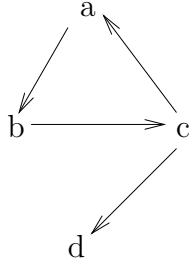


Figure 4: The graph for for prog4

```

arc(c,a).          % clause 5
arc(c,d).          % clause 6
  
```

The graph described by clauses 3,4,5,6 of prog4 is displayed in Figure 4. Now let us compile this program and then ask the query below.

```
?- path(a,d).
```

We would expect a true answer because there is a path from a to d, namely $a \rightarrow b \rightarrow c \rightarrow d$.

However, the answer is

```
ERROR: Out of local stack
```

which indicates an infinite loop.

So, let us trace the program, as shown in Figure 5. We see that the leaf $\text{path}(a,d)$ is the same as the root, so we have a non-terminating loop. So, the loop in the graph, caused a loop in the program!

3. The path2 predicate

One way of avoiding this loop is to rewrite the path predicate. Let us place the recursive call in the second clause in front the arc goal. We have prog5.

```

% define the path2 based on the arc predicate.
path2(X,X).          % clause 1
path2(X,Y) :- path2(X,Z), arc(Z,Y).          % clause 2
% the arc clauses
arc(a,b).            % clause 3
arc(b,c).            % clause 4
arc(c,a).            % clause 5
arc(c,d).            % clause 6
  
```

After we consult prog5, the query

```
?- path2(a,d).
```

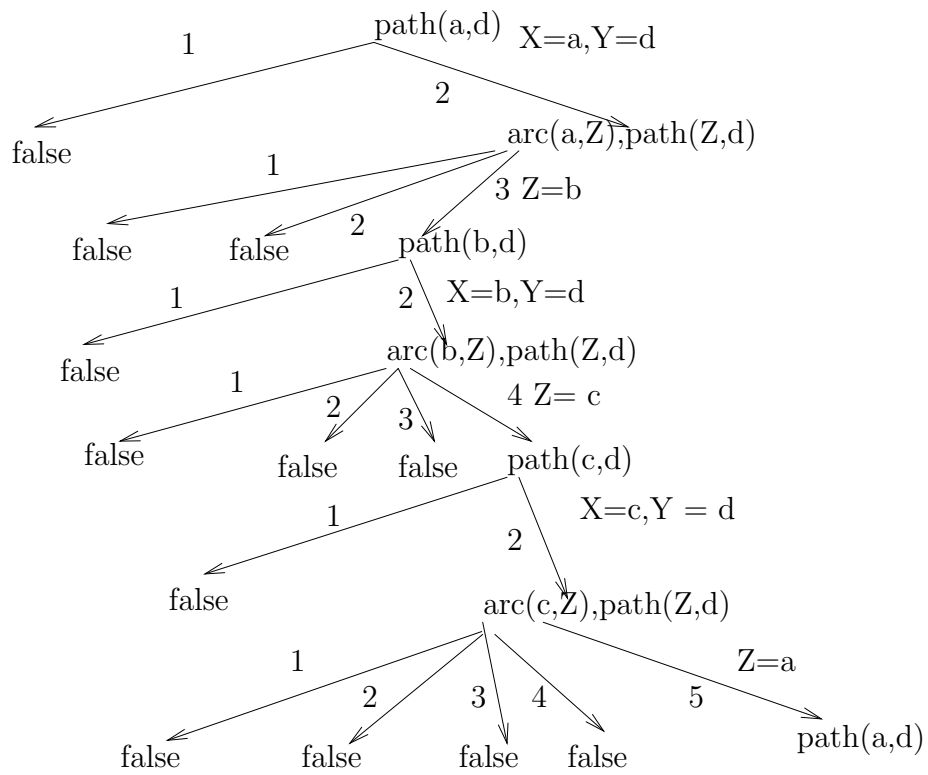


Figure 5: The trace for the query path(a,d) for prog4

produces the expected answer,
true.

Figure 6 traces the execution of the program.

From Figure 6 we can see how `path2` works. It first generates the goal `arc(a,d)`. If this fails, it generates the goal `arc(a,Z),arc(Z,d)`. If this fails, it produces the goal `arc(a,Z1),arc(Z1,Z),arc(Z,d)`.

So, for the goal `path2(vertex1,vertex2)`, `path2` will first check for a path of length 0, i.e. `vertex1 = vertex2`. If this fails, it will check if there is path of length 1 from `vertex1` to `vertex2`. If this is not possible, it will try the paths of length 2, 3, 4, and so on. So, if there is finite path from `vertex1` to `vertex2`, `path2(vertex1,vertex2)` will find it.

What is the drawback of this predicate? If there is no path from `vertex1` to `vertex2`, `path2` will try all paths of lengths 0, 1, 2, 3, and so on as before. Except that it will never stop. This is true even if there is no arrow coming out of `vertex1`.

We will write a better path predicate after we study lists.

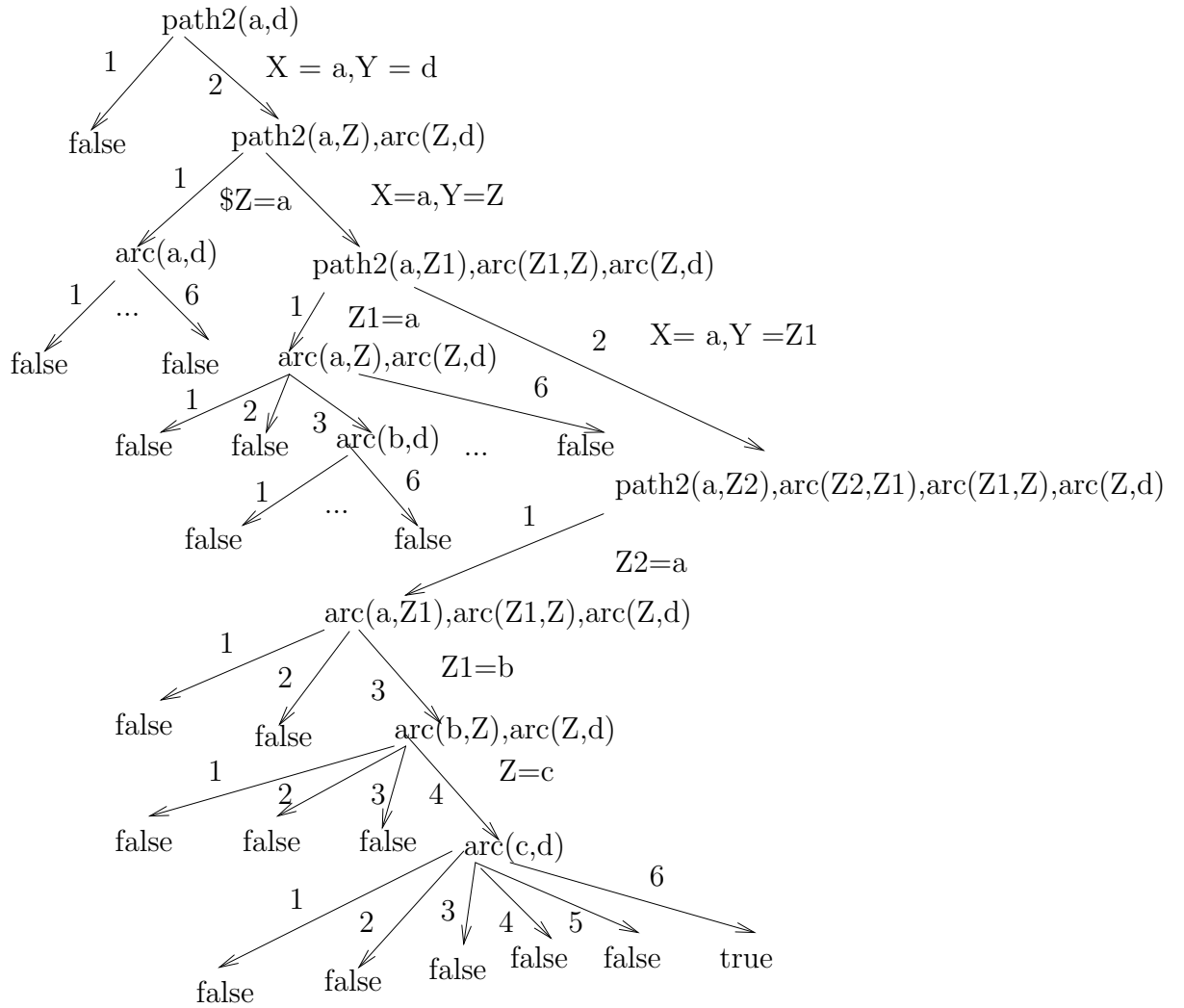


Figure 6: The trace for the query $\text{path2}(a,d)$ for prog5