COT 3541
Section U03
Spring 2017

THE FINAL EXAM ANSWERS

Question 1. (15 points)

$F \equiv \forall x \exists y_1 [P(x, y_1) \vee \exists x_1 (\neg Q(x_1, y_1) \wedge \forall y_2 P(y_2, x_1))] \wedge \forall z (\forall x_2 R(x_2, y, z) \vee \exists z_1 \exists x_3 \neg Q(x_3, z_1, x_3))$

Grading Criteria:

-1 point for each error, up to 13.

Question 2. (20 points)

Find a resolution tree of $\Box$ from

$S = \{\{P(x, y, f(z)), Q(x, y)\}, \{\neg P(g(x), x, y), \neg P(z, u, f(v)), Q(z, v)\},$
$\{\neg R(a, f(x)), \neg Q(g(x), x)\}, \{R(y, x), R(a, f(y)), \neg Q(u, y)\}\}.$

Draw the tree on a blank sheet of paper. For each step specify the relabelings, the unifying set, the mgu, and the resolvent.

Solution: The answer is in Figure 1.

Grading Criteria: 1. 6.5 points for each resolution step: 0.75 point for the subs, 0.75 points for S, 2 points for σ, 3 points for the resolvent.

2. -3 points for each extra resolution step.

3. If one of the parents is wrong you loose an additional 3.5 points for the step.

4. If both parents are wrong, there is no credit for the step.

Question 3. (15 points)

Solution: $D(F, 1) = \{a, g(a), h(a)\}$.

$E(F, 1) = \{F^M[x/t_1, y/t_2] | t_1, t_2 \in D(F, 1)\}$
$= \{F^M[x/a, y/a], F^M[x/a, y/g(a)], F^M[x/a, y/h(a)],$
$F^M[x/g(a), y/a], F^M[x/g(a), y/g(a)], F^M[x/g(a), y/h(a)],$
$F^M[x/h(a), y/a], F^M[x/h(a), y/g(a)], F^M[x/h(a), y/h(a)]\}$

Grading Criteria: -1.5 point for each missimg, wrong, or extra formula.

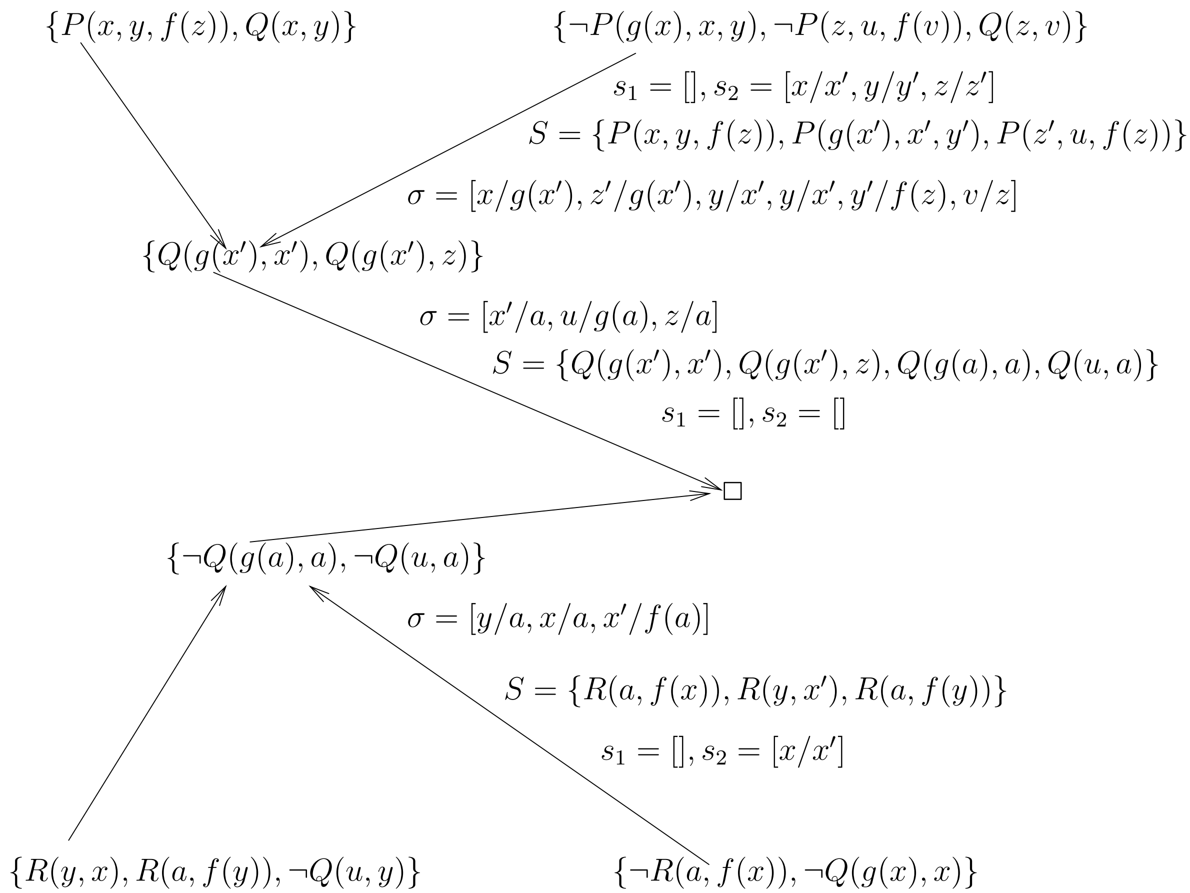

Figure 1: The resolution for Question 2

Question 4. (15 points)
1. 3 2. 4 3. 0 4. 1 5. 1

Grading Criteria: 3 points for each correct answer.

Question 5. (12 points)

1. 3 points for each step
2. 2 points for trying

Proof:

$(\forall x F \longrightarrow G) \equiv \neg \forall x F \vee G$ $\longrightarrow$-elim
$\equiv \exists x \neg F \vee G$ push $\neg$ past $\forall$
$\equiv \exists x(\neg F \vee G)$ since x is not free in G we can pull out $\exists x$
$\equiv \exists x(F \longrightarrow G)$ $\longrightarrow$-intro

Grading Criteria: 1. Skipping steps : - 3 points per step
Question 6. (20 points)
Proof: Let $\mathcal{A}$ be a structure with universe U. We assume (1) and (2) and need to prove (3).

(1) $\mathcal{A}[(F \vee G)[x/t]] = 1$
(2) $\mathcal{A}[\forall x \neg G] = 1$
(3) $\mathcal{A}[\exists x F] = 1$

Since t is free for x in $(F \vee G)$, we can apply the translation lemma to $\mathcal{A}[(F \vee G)[x/t]]$ and get (4).

(4) $\mathcal{A}[(F \vee G)[x/t]] = \mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F \vee G]$
From (1) and (4) we get (5).
(5) $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F \vee G] = 1$
We remove the quantifier from (2) and get that
$\mathcal{A}_{[x \leftarrow d]}[\neg G] = 1$
for all $d \in U$. We choose $d = t^{\mathcal{A}}$ and get (6).
(6) $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[\neg G] = 1$
We remove the $\neg$ from (6) and have (7).
(7) $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[G] = 0$
Now we compute $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F \vee G]$.
$\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F \vee G]$
$= \mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F] \vee \mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[G]$ by the interpretation Of $\vee$
$= \mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F] \vee 0$ by (7)

So, we derived (8).
(8) $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F \vee G] = \mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F]$
From (5) and (8) we get (9).
(9) $\mathcal{A}_{[x \leftarrow t^{\mathcal{A}}]}[F] = 1$
From (9) we get (3) by the interpretation of $\exists x$.

Grading Criteria:
1. 2.5 points for trying

Question 7. (12 points)

```
?- print([1,2,3,4,5]).
2
4
5
3
1
true

?- path(a,d).
true
?- path(d,a).
ERROR: Out of local stack

?- loves(X,Y), loves(Y,X).
X = john,
Y = mary
```

Grading Criteria: 5 points for the first query, 2 for the second, 2 for the third and 3 for the third
1.5 points for trying

Question 8. (18 points)

a. (8 points)

```
% isIntegerList(L) is satisfied if L is a list of integers
isIntegerList([]).          % [] is an integer list
isIntegerList([Head | Tail]) :- integer(Head), isIntegerList(Tail).
```

Grading Criteria

1. 3 points for the base case
2. 5 points for the recursion
3. 1 point for trying

b. (12 points)

```
% listSum(L,S) is satisfied if S is the sum of all integers in the list L
listSum([X],X) :- integer(X).
listSum([Head | Tail], M) :-listSum(Tail,N), integer(Head), !, M is N + Head.
listSum([Head | Tail], M) :- listSum(Tail,M).
```

Grading Criteria

1. 3 points for the base case
2. 9 points for the recursion
3. 1.5 points for trying