**COT 3541**
**Section U01**
**Spring 2017**

## MIDTERM EXAM ANSWERS

## QUESTIONS

**Question 1.** (15 points)
1. a 2. a 3. c 4. a 5. c 6. c 7. a 8. d 9. c 10. c 11. c 12. b 13. c 14. c 15. c

**Grading Criteria:** 1 point for each correct answer.

**Question 2.** (15 points)
Prove, by structural induction on $F$, that every non-empty suffix $S$ of $F$ with $n[(,S] = n[),S]$ is a formula.
Write your proof on a blank sheet of paper.

**Solution**
We need to prove $\mathbf{P}[F]$.
$\mathbf{P}[F]$ Every nonempty suffix of $F$ with $n[(,S] = n[),S]$ is a subformula of $F$.
So we have to show that every $S$ satisfies $(*)$.
(*) if $S$ is a nonempty suffix of $F$ with $n[(,S] = n[),S]$ is a subformula of $F$.
**Case 1:** $F$ is an atom.
The only suffixes of $F$ are $S = \lambda$ and $S = F$. If $S = \lambda$, the statement is vacuously true. If $S = F$, $S$ is a formula, so again $(*)$ is true.
**Case 2:** $F = \neg G$.
The suffixes of $F$ are either suffixes of $G$ or $F$ itself. If $S$ is a suffix of $G$, then $S$ satisfies $(*)$ by IH. If $S = F$, then $(*)$ is satisfied because $S$ is a formula.
**Cases 3,4,5,6:** $F = (GCH)$ where $C$ is a binary connective.
The suffixes of $F$ fall into 4 subcase below:
Subcase 3.1: $S = \lambda$.

Then $(*)$ is vacuously true.

Subcase 3.2: $S = J)$ where $J$ is a suffix of $H$.

$n[), S] = n[), J] + 1$ because $S = J)$

$> n[), J]$ drop the 1

$\geq n[(, J]$ by Lemma 2 from the class

$= n[(, J)]$

$= n[(, S]$ because $S = J)$

So, $n[), S] > n[(, S]$.

For these suffixes $S$, $(*)$ is vacuously true.

Subcase 3.3: $S = ICH)$ where $I$ is a suffix of $G$.

$n[), S] = n[), I] + n[), H] + 1$ because $S = ICH)$

$> n[), I] + n[), H]$ drop the 1

$\geq n[(, I] + n[(, H]$ Lemma 2 from the class notes applied twice

$= n[(, ICH)$

$= n[(, S]$ because $S = ICH)$

So, $n[), S] > n[(, S]$.

For these suffixes $S$, $(*)$ is vacuously true.

Subcase 3.4: $S = F$. Since $S$ is a formula, $S$ satisfies $(*)$.

**Grading Criteria:**

1. Listing the cases: 2 points
2: Case 1: 2 points
3. Case 2: 2 points
4. Cases 3,4,5,6: 9 points
    4.1 the split into subcases and Subcase 3.1: 2 points
    4.2 Subcase 3.2: 3 points
    4.2 Subcase 3.3: 3 points
    4.3 Subcase 3.4: 1 point

**Question 3.** (15 points)

Apply the algorithm given in the book to find a CNF for the formula
$F = \neg[(A \vee B \vee \neg C) \longleftrightarrow \neg(\neg B \wedge C \wedge \neg D)]$.

**Solution**

$F = \neg[(A \vee B \vee \neg C) \longleftrightarrow \neg(\neg B \wedge C \wedge \neg D)]$ line 1

$\equiv \neg\{[(A \vee B \vee \neg C) \longrightarrow \neg(\neg B \wedge C \wedge \neg D)] \wedge [\neg(\neg B \wedge C \wedge \neg D) \longrightarrow (A \vee B \vee \neg C)]\}$ $\longleftrightarrow$-elim, line 2

$\equiv \neg\{[\neg(A \vee B \vee \neg C) \vee \neg(\neg B \wedge C \wedge \neg D)] \wedge [\neg\neg(\neg B \wedge C \wedge \neg D) \vee A \vee B \vee \neg C]\}$
$\longrightarrow$-elim twice, line 3

$\equiv \neg[\neg(A \vee B \vee \neg C) \vee \neg(\neg B \wedge C \wedge \neg D)] \vee \neg[\neg\neg(\neg B \wedge C \wedge \neg D) \vee A \vee B \vee \neg C]$
DeMorgan's, line 4

$\equiv [\neg\neg(A \vee B \vee \neg C) \wedge \neg\neg(\neg B \wedge C \wedge \neg D)] \vee [\neg\neg\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge \neg\neg C]$ DeMorgan's, generalized DeMorgan's, line 5

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge C]$
$\neg\neg$-elim 4 times, line 6

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge C]$
$\neg\neg$-elim 4 times, line 7

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [(\neg\neg B \vee \neg C \vee \neg\neg D) \wedge \neg A \wedge \neg B \wedge C]$
DeMorgan's, line 8

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [(B \vee \neg C \vee D) \wedge \neg A \wedge \neg B \wedge C]$
$\neg\neg$-elim 2 times, line 9

$\equiv (A \vee B \vee \neg C \vee B \vee \neg C \vee D) \wedge (A \vee B \vee \neg C \vee \neg A) \wedge (A \vee B \vee \neg C \vee \neg B) \wedge (A \vee B \vee \neg C \vee C) \wedge (\neg B \vee B \vee \neg C \vee D) \wedge (\neg B \vee \neg A) \wedge (\neg B \vee \neg B) \wedge (\neg B \vee C) \wedge (C \vee B \vee \neg C \vee D) \wedge (C \vee \neg A) \wedge (C \vee \neg B) \wedge (C \vee C) \wedge (\neg D \vee B \vee \neg C \vee D) \wedge (\neg D \vee \neg A) \wedge (\neg D \vee \neg B) \wedge (\neg D \vee C)$ Generalized distributivity, line 10

$\equiv (A \vee B \vee \neg C \vee B \vee \neg C \vee D) \wedge (\neg B \vee \neg A) \wedge (\neg B \vee \neg B) \wedge (\neg B \vee C) \wedge (C \vee \neg A) \wedge (C \vee \neg B) \wedge (C \vee C) \wedge (\neg D \vee \neg A) \wedge (\neg D \vee \neg B) \wedge (\neg D \vee C)$
tautology elim, line 11

$\equiv (A \vee B \vee \neg C \vee D) \wedge (\neg A \vee \neg B) \wedge \neg B \wedge (\neg B \vee C) \wedge (\neg A \vee C) \wedge (\neg B \vee C) \wedge C \wedge (\neg A \vee \neg D) \wedge (\neg B \vee \neg D) \wedge (C \vee \neg D)$ idempotency, ordering the clauses, line 12

$\equiv (A \vee B \vee \neg C \vee D) \wedge \neg B \wedge C \wedge (\neg A \vee \neg D)$ absorbtion, line 13

**Note** If we use the reduction $(F \vee \neg G) \wedge G \implies F \wedge G$, then the above formula reduces to $(A \vee D) \wedge \neg B \wedge C \wedge (\neg A \vee \neg D)$ and we can simplify the computation quite a lot. For example, $[(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)]$ reduces to $[A \wedge \neg B \wedge C \wedge \neg D)]$ and $[(B \vee \neg C \vee D) \wedge \neg A \wedge \neg B \wedge C]$ reduces to $[D \wedge \neg A \wedge \neg B \wedge C]$.

**Grading Criteria:** You get credit to the first line where you made a mistake.

1. If you got the first line correct you have 1 point.
2. If you got the first 2 lines correct you have 2 points.
3. If you got the first 3 lines correct you have 3 points.
4. If you got the first 4 lines correct you have 4 points.
5. If you got the first 5 lines correct you have 5 points.

6. If you got the first 6 lines correct you have 6 points.
7. If you got the first 7 lines correct you have 7 points.
8. If you got the first 8 lines correct you have 8 points.
9. If you got the first 9 lines correct you have 9 points.
10. If you got the first 10 lines correct you have 10 points.
11. If you got the first 11 lines correct you have 12 points.
12. If you got the first 12 lines correct you have 13 points.
13. If you got all 13 lines correct you have 15 points.
14. Not separating formulas by $\equiv$ : -2 points
15. Using parentheses around atoms or negations: -2 points
16. Not writing what rules are applied to get the equivalences : -2 points

**Question 4.** (10 points)
**Answers:** 1. c 2. a, d 3. d 4. a 5. c 6. b 7. c 8. c 9. b 10. c
**Grading Criteria:** 1 point for each correct answer.

**Question 5.** (15 points)
Construct a derivation tree of $\Box$ from
$S = \{\{\neg A, \neg B, \neg C, \neg D\}, \{\neg A, C\}, \{\neg A, \neg C, D\}, \{A\}, \{\neg A, B\}\}$.

The answer is in Figure 1.

**Grading Criteria:** 1. 3.75 points for each correct resolution step (up to 4) that leads to $\Box$.
2. If one parent is wrong (either was not in the set or was the result of a wrong resolution), you loose 2 points.
3. -4 points for each incorrect resolution step.
4. - 1.5 points for each extra or useless resolution step.
5. 2 points for trying.

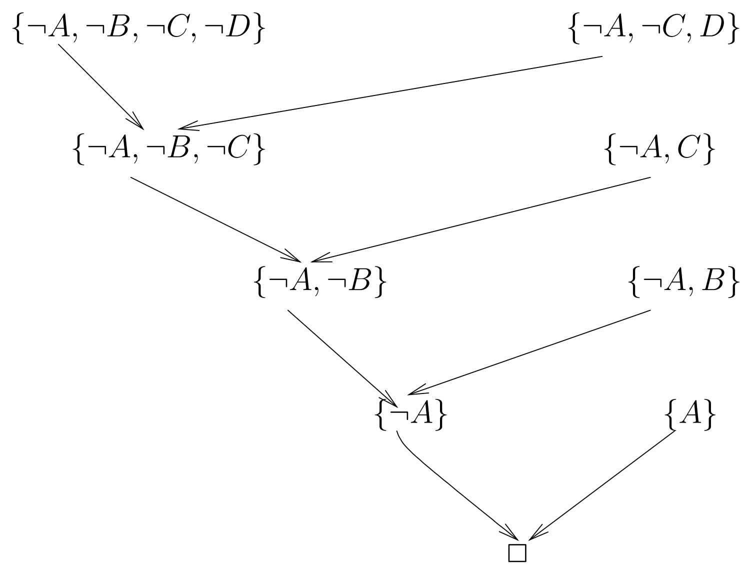


Figure 1: The resolution tree for Question 5