COT 3541
Section U03
Spring 2017

MIDTERM EXAM ANSWERS

Question 1. (15 points)

1. b 2. c 3. a 4. c 5. a 6. c 7. c 8. b 9. c 10. b 11. c 12. d 13. c 14. d 15. c

Grading Criteria: 1 points for each correct answer.

Question 2. (15 points)
Case 1: $F = P_i$ for some $i \in N$. Since P_i is an atom, $|F| = 1$ and $n[con, F] = n[\neg, F] = 0$. The equality becomes

$1 = 4 * 0 + 0 + 1$

which is true.
Case 2: $F = \neg G$.

By IH,

(IH) $|G| = 4 * n[con, G] + n[\neg, G] + 1$

Now we relate the length and the F-counts to the length and the G-counts.

(1) $|F| = |G| + 1$
(2) $n[con, F] = n[con, G]$
(3) $n[\neg, F] = n[\neg, G] + 1$

Now,

$|F| = |G| + 1$ by (1)
$= (4 * n[con, G] + n[\neg, G] + 1) + 1$ by (IH)
$= 4 * n[con, G] + (n[\neg, G] + 1) + 1$ by grouping
$= 4 * n[con, F] + (n[\neg, G] + 1) + 1$ by (2)
$= 4 * n[con, F] + n[\neg, F] + 1$ by (3)

So we got $|F| = 4 * n[con, F] + n[\neg, F] + 1$.

Case 3: $F = (GCH)$ where C is a binary connective.

By IH on G and H we have:

(IH1) $|G| = 4 * n[con, G] + n[\neg, G] + 1$

(IH2) $|H| = 4 * n[con, H] + n[\neg, h] + 1$

Now we relate the length and the F-counts to the lengths and the counts of G and H.

(4) $|F| = |G| + |H| + 3$

(5) $n[con, F] = n[con, G] + n[con, H] + 1$

(6) $n[\neg, F] = n[\neg, G] + n[\neg, H]$

Now we relate $|F|$ to the F counts.

$|F| = |G| + |H| + 3$ by (4)

$= (4 * n[con, G] + n[\neg, G] + 1) + |H| + 3$ by (IH1)

$= (4 * n[con, G] + n[\neg, G] + 1) + (4 * n[con, H] + n[\neg, H] + 1) + 3$ by (IH2)

$= (4*n[con, G]+4*n[con, H]+4)+(n[\neg, G]+n[\neg, H])+1$ by grouping

$= 4*(n[con, G]+n[con, H]+1)+(n[\neg, G]+n[\neg, H])+1$ by distributivity

$= 4 * n[con, F] + (n[\neg, G] + n[\neg, H]) + 1$ by (5)

$= 4 * n[con, F] + n[\neg, F] + 1$ by (6)

So we got $|F| = 4 * n[con, F] + n[\neg, F] + 1$.

Grading Criteria:

1. Listing the cases: 2 points
2. Case 1: 1 points
3. Case 2: 4 points
   - 3.1: the IH: 1 point
   - 3.2: formulas (1)-(3): 1.5 points
   - 3.3: the derivation: 1.5 points
4. Cases 3-6: 8 points
   - 4.1: the IH: 1.5 points
   - 4.2: formulas (4)-(6): 1.5 points
   - 4.3: the derivation: 4 points
     - 4.3.1: the explanation of the derivation: 1 point
5. Bad style, like omissions, not labeling the formulas, not indicating the proof method: -2 points

Question 3. (15 points)

Step 1. Eliminate $\longleftrightarrow$'s.

$F = \neg[(A \wedge \neg(B \vee C)) \longleftrightarrow \neg(B \vee D)]$.

$\equiv \neg\{[(A \wedge \neg(B \vee C)) \longrightarrow \neg(B \vee D)] \wedge [\neg(B \vee D) \longrightarrow (A \wedge \neg(B \vee C))]\}$

Step 2. Eliminate $\longrightarrow$'s.

$\equiv \neg\{[\neg(A \wedge \neg(B \vee C)) \vee \neg(B \vee D)] \wedge [\neg\neg(B \vee D) \vee (A \wedge \neg(B \vee C))]\}$

Step 3. Push the negation inside.

$\equiv \neg[\neg(A \wedge \neg(B \vee C)) \vee \neg(B \vee D)] \vee \neg[\neg\neg(B \vee D) \vee (A \wedge \neg(B \vee C))]$ // De Morgan's law

$\equiv [\neg\neg(A \wedge \neg(B \vee C)) \wedge \neg\neg(B \vee D)] \vee [\neg\neg\neg(B \vee D) \wedge \neg(A \wedge \neg(B \vee C))]$ // De Morgan's law twice

$\equiv [(A \wedge \neg(B \vee C)) \wedge (B \vee D)] \vee [\neg(B \vee D) \wedge (\neg A \vee \neg\neg(B \vee C))]$ // double $\neg$- elim 3 times, De Morgan's law

$\equiv [A \wedge \neg B \wedge \neg C \wedge (B \vee D)] \vee [\neg B \wedge \neg D \wedge (\neg A \vee B \vee C)]$ // De Morgan's law twice, double neg elim

Step 4. Apply the distributivity.

$\equiv (A \vee \neg B) \wedge (A \vee \neg D) \wedge (A \vee \neg A \vee B \vee C) \wedge$
$(\neg B \vee \neg B) \wedge (\neg B \vee \neg D) \wedge (\neg B \vee \neg A \vee B \vee C) \wedge$
$(\neg C \vee \neg B) \wedge (\neg C \vee \neg D) \wedge (\neg C \vee \neg A \vee B \vee C) \wedge$
$(B \vee D \vee \neg B) \wedge (B \vee D \vee \neg D) \wedge (B \vee D \vee \neg A \vee B \vee C)$ // Generalized distributivity

Step 5. Simplify

$\equiv (A \vee \neg B) \wedge (A \vee \neg D) \wedge (\neg B \vee \neg B) \wedge (\neg B \vee \neg D) \wedge$
$(\neg C \vee \neg B) \wedge (\neg C \vee \neg D) \wedge (B \vee D \vee \neg A \vee B \vee C)$ // tautology elimination

$\equiv (A \vee \neg B) \wedge (A \vee \neg D) \wedge \neg B \wedge (\neg B \vee \neg D) \wedge$
$(\neg B \vee \neg C) \wedge (\neg C \vee \neg D) \wedge (\neg A \vee B \vee C \vee D)$ // idempotency, ordering the literals

$\equiv (A \vee \neg D) \wedge \neg B \wedge (\neg C \vee \neg D) \wedge (\neg A \vee B \vee C \vee D)$ // absorbtion

Grading Criteria:

You get credit up to the first error.

1. Step 1: 1.5 points
2. Step 2: 1.5 points
3. Step 3: 5.5 points
4. Step 4: 4 points
5. Step 5: 2.5 points
6. No reasons : -4 points

Question 4. (10 points)

1. b 2. c 3. b 4. d 5. c 6. a 7. d 8. a 9. c 10. b

Grading Criteria: 1 points for each correct answer.

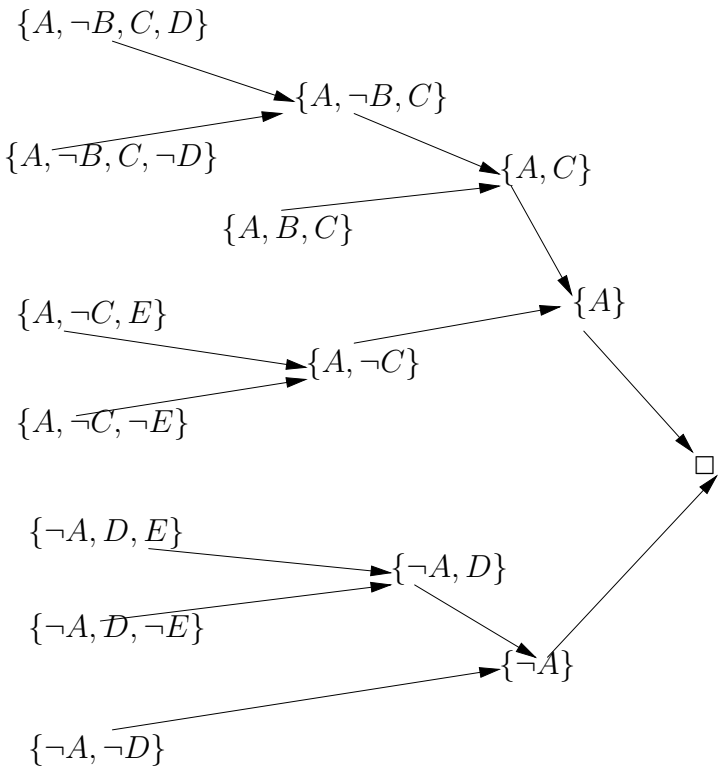

Figure 1: The Derivation Tree for Question 5

Question 5. (15 points)
Construct a derivation tree of $\square$ from
$S = \{\{A, B, C\}, \{A, \neg B, C, D\}, \{A, \neg B, C, \neg D\}, \{A, \neg C, \neg E\}, \{A, \neg C, E\}, \{\neg A, \neg D\}, \{\neg A, D, E\}, \{\neg A, D, \neg E\}\}$.

Answer: The derivation tree is shown in Figure 1.
Grading Criteria: 1. 2 for each correct resolution leading to $\square$.
2. -2 points for each incorrect derivation step.
3. -2 points for diagrams that are hard to follow.
4. If one parent is wrong you loose 1 point even if the derivation is correct.
5. Score = number of correct resolvents - penalties