COT 3541
Section U02
Fall 2016

MIDTERM EXAM ANSWERS

QUESTIONS

Question 1. (15 points)

1. a 2. a 3. c 4. a 5. c 6. c 7. a 8. d 9. c 10. c 11. c 12. b 13. c 14. c 15. c

Grading Criteria: 1 point for each correct answer.

Question 2. (15 points)

Case 1: $F = P_i$ for some $i \in N$. Since P_i is an atom, $|F| = 1$ and $n[con, F] = n[\neg, F] = 0$. The equality becomes

$1 = 4 * 0 + 0 + 1$

which is true.

Case 2: $F = \neg G$.

By IH,

(IH) $|G| = 4 * n[con, G] + n[\neg, G] + 1$

Now we relate the length and the F-counts to the length and the G-counts.

(1) $|F| = |G| + 1$

(2) $n[con, F] = n[con, G]$

(3) $n[\neg, F] = n[\neg, G] + 1$

Now,

$|F| = |G| + 1$ by (1)

$= (4 * n[con, G] + n[\neg, G] + 1) + 1$ by (IH)

$= 4 * n[con, G] + (n[\neg, G] + 1) + 1$ by grouping

$= 4 * n[con, F] + (n[\neg, G] + 1) + 1$ by (2)

$= 4 * n[con, F] + n[\neg, F] + 1$ by (3)

So we got $|F| = 4 * n[con, F] + n[\neg, F] + 1$.

Case 3: $F = (GCH)$ where C is a binary connective.

By IH on G and H we have:

(IH1) $|G| = 4 * n[con, G] + n[\neg, G] + 1$

(IH2) $|H| = 4 * n[con, H] + n[\neg, h] + 1$

Now we relate the length and the F-counts to the lengths and the counts of G and H.

(4) $|F| = |G| + |H| + 3$

(5) $n[con, F] = n[con, G] + n[con, H] + 1$

(6) $n[\neg, F] = n[\neg, G] + n[\neg, H]$

Now we relate $|F|$ to the F counts.

$|F| = |G| + |H| + 3$ by (4)

$= (4 * n[con, G] + n[\neg, G] + 1) + |H| + 3$ by (IH1)

$= (4 * n[con, G] + n[\neg, G] + 1) + (4 * n[con, H] + n[\neg, H] + 1) + 3$ by (IH2)

$= (4*n[con, G] + 4*n[con, H] + 4) + (n[\neg, G] + n[\neg, H]) + 1$ by grouping

$= 4*(n[con, G] + n[con, H] + 1) + (n[\neg, G] + n[\neg, H]) + 1$ by distributivity

$= 4 * n[con, F] + (n[\neg, G] + n[\neg, H]) + 1$ by (5)

$= 4 * n[con, F] + n[\neg, F] + 1$ by (6)

So we got $|F| = 4 * n[con, F] + n[\neg, F] + 1$.

Grading Criteria:

1. Listing the cases: 2 points
2. Case 1: 1 points
3. Case 2: 4 points
   - 3.1: the IH: 1 point
   - 3.2: formulas (1)-(3): 1.5 points
   - 3.3: the derivation: 1.5 points
4. Cases 3-6: 8 points
   - 4.1: the IH: 1.5 points
   - 4.2: formulas (4)-(6): 1.5 points
   - 4.3: the derivation: 4 points
     - 4.3.1: the explanation of the derivation: 1 point
5. Bad style, like omissions, not labeling the formulas, not indicating the proof method: -2 points

Question 3 (15 points)

Apply the algorithm given in the book to find a CNF for the formula

$F = \neg[(A \vee B \vee \neg C) \longleftrightarrow \neg(\neg B \wedge C \wedge \neg D)]$.

Solution

$F = \neg[(A \vee B \vee \neg C) \longleftrightarrow \neg(\neg B \wedge C \wedge \neg D)]$ line 1

$\equiv \neg\{[(A \vee B \vee \neg C) \longrightarrow \neg(\neg B \wedge C \wedge \neg D)] \wedge [\neg(\neg B \wedge C \wedge \neg D) \longrightarrow (A \vee B \vee \neg C)]\}$ $\longleftrightarrow$-elim, line 2

$\equiv \neg\{[\neg(A \vee B \vee \neg C) \vee \neg(\neg B \wedge C \wedge \neg D)] \wedge [\neg\neg(\neg B \wedge C \wedge \neg D) \vee A \vee B \vee \neg C]\}$ $\longrightarrow$-elim twice, line 3

$\equiv \neg[\neg(A \vee B \vee \neg C) \vee \neg(\neg B \wedge C \wedge \neg D)] \vee \neg[\neg\neg(\neg B \wedge C \wedge \neg D) \vee A \vee B \vee \neg C]$ DeMorgan's, line 4

$\equiv [\neg\neg(A \vee B \vee \neg C) \wedge \neg\neg(\neg B \wedge C \wedge \neg D)] \vee [\neg\neg\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge \neg\neg C]$ DeMorgan's, generalized DeMorgan's, line 5

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge C]$ $\neg\neg$-elim 4 times, line 6

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [\neg(\neg B \wedge C \wedge \neg D) \wedge \neg A \wedge \neg B \wedge C]$ $\neg\neg$-elim 4 times, line 7

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [(\neg\neg B \vee \neg C \vee \neg\neg D) \wedge \neg A \wedge \neg B \wedge C]$ DeMorgan's, line 8

$\equiv [(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)] \vee [(B \vee \neg C \vee D) \wedge \neg A \wedge \neg B \wedge C]$ $\neg\neg$-elim 2 times, line 9

$\equiv (A \vee B \vee \neg C \vee B \vee \neg C \vee D) \wedge (A \vee B \vee \neg C \vee \neg A) \wedge (A \vee B \vee \neg C \vee \neg B) \wedge (A \vee B \vee \neg C \vee C) \wedge (\neg B \vee B \vee \neg C \vee D) \wedge (\neg B \vee \neg A) \wedge (\neg B \vee \neg B) \wedge (\neg B \vee C) \wedge (C \vee B \vee \neg C \vee D) \wedge (C \vee \neg A) \wedge (C \vee \neg B) \wedge (C \vee C) \wedge (\neg D \vee B \vee \neg C \vee D) \wedge (\neg D \vee \neg A) \wedge (\neg D \vee \neg B) \wedge (\neg D \vee C)$ Generalized distributivity, line 10

$\equiv (A \vee B \vee \neg C \vee B \vee \neg C \vee D) \wedge (\neg B \vee \neg A) \wedge (\neg B \vee \neg B) \wedge (\neg B \vee C) \wedge (C \vee \neg A) \wedge (C \vee \neg B) \wedge (C \vee C) \wedge (\neg D \vee \neg A) \wedge (\neg D \vee \neg B) \wedge (\neg D \vee C)$ tautology elim, line 11

$\equiv (A \vee B \vee \neg C \vee D) \wedge (\neg A \vee \neg B) \wedge \neg B \wedge (\neg B \vee C) \wedge (\neg A \vee C) \wedge (\neg B \vee C) \wedge C \wedge (\neg A \vee \neg D) \wedge (\neg B \vee \neg D) \wedge (C \vee \neg D)$ idempotency, ordering the clauses, line 12

$\equiv (A \vee B \vee \neg C \vee D) \wedge \neg B \wedge C \wedge (\neg A \vee \neg D)$ absorbtion, line 13

Note If we use the reduction $(F \vee \neg G) \wedge G \Longrightarrow F \wedge G$, then the above formula reduces to $(A \vee D) \wedge \neg B \wedge C \wedge (\neg A \vee \neg D)$ and we can simplify the computation quite a lot. For example, $[(A \vee B \vee \neg C) \wedge \neg B \wedge C \wedge \neg D)]$ reduces to $[A \wedge \neg B \wedge C \wedge \neg D)]$ and $[(B \vee \neg C \vee D) \wedge \neg A \wedge \neg B \wedge C]$ reduces to $[D \wedge \neg A \wedge \neg B \wedge C]$.

Grading Criteria: You get credit to the first line where you made a mistake.

1. If you got the first line correct you have 1 point.

2. If you got the first 2 lines correct you have 2 points.
3. If you got the first 3 lines correct you have 3 points.
4. If you got the first 4 lines correct you have 4 points.
5. If you got the first 5 lines correct you have 5 points.
6. If you got the first 6 lines correct you have 6 points.
7. If you got the first 7 lines correct you have 7 points.
8. If you got the first 8 lines correct you have 8 points.
9. If you got the first 9 lines correct you have 9 points.
10. If you got the first 10 lines correct you have 10 points.
11. If you got the first 11 lines correct you have 12 points.
12. If you got the first 12 lines correct you have 13 points.
13. If you got all 13 lines correct you have 15 points.
14. Not separating formulas by $\equiv$: -2 points
15. Using parentheses around atoms or negations: -2 points
16. Not writing what rules are applied to get the equivalences : -2 points

Question 4. (10 points)

1. b 2. c 3. d 4. b 5. c 6. d 7. d 8. b 9. a 10. c

Grading Criteria: 1 point for each correct answer.

Question 5. (15 points)

Construct a derivation tree of $\square$ from
$S = \{\{\neg A, \neg B, \neg C, \neg D\}, \{\neg A, C\}, \{\neg A, \neg C, D\}, \{A\},$
$\{\neg A, B\}\}$.

The answer is in Figure 1.

Grading Criteria: 1. 3.75 points for each correct resolution step (up to 4) that leads to $\square$.

2. -4 points for each incorrect resolution step.
3. - 1.5 points for each extra or useless resolution step.
4. 2 points for trying.

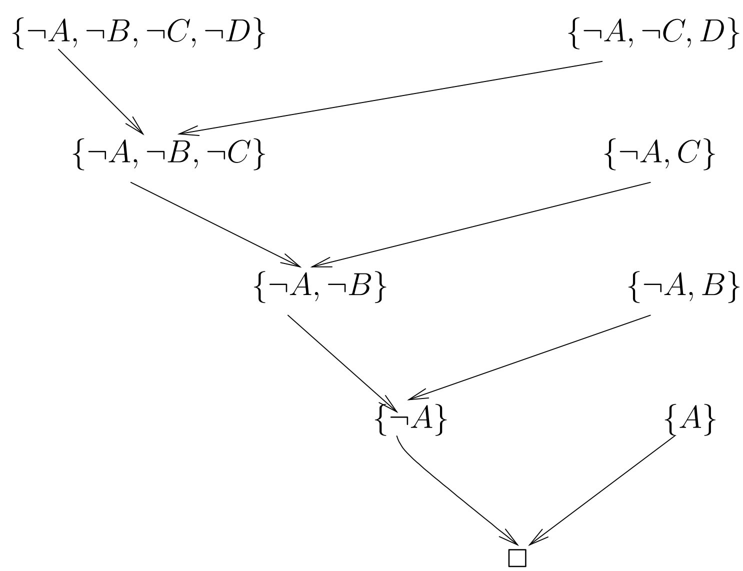

Figure 1: The resolution tree for Question 5