

COT 3541
Section U03
Spring 2017

FINAL EXAM

INSTRUCTIONS

1. This test is closed book, closed notebook.
2. You cannot use computers and phones.
3. There are 8 questions for a total of 130 points.
4. Write the answers to questions 1, 3, 4, 7 and 8 on the exam paper.
Write the answers to the other questions on the blank sheets.
5. If you do not understand the meaning of a question ask me during the test.
6. You have 2 hours to work on the test.
7. If you need to go to the bathroom, do it before you start the exam.
8. Have a nice vacation!
9. Write your name below.

NAME: -----

QUESTIONS

Question 1. (15 points)

Rectify the formula F .

$$F = \forall x \exists y [P(x, y) \vee \exists x (\neg Q(x, y) \wedge \forall y P(y, x))] \wedge \forall z (\forall x R(x, y, z) \vee \exists z \forall y \exists x \neg Q(x, z, x))$$

Write your answer below.

Question 2. (20 points)

Draw the tree on a blank sheet of paper. For each step specify the relabelings, the unifying set, the mgu, and the resolvent.

Derive the box from the set $S = \{\{P(x, y, z), Q(x, y)\}, \{\neg P(a, f(x), y), \neg P(x, y, f(z)), Q(x, u)\}, \{R(x, g(x, y)), \neg Q(x, y)\}, \{\neg R(a, x), \neg R(y, g(a, f(y)))\}\}$

Draw your tree on a blank sheet of paper. For each derivation step specify the relabelings, the unifying set, the mgu and the resolvent.

Question 3. (15 points)

Write $E(F, 1)$ for $F = \forall x \forall y [P(a, g(x)) \wedge \neg P(y, h(x)) \wedge \neg P(h(a), y)]$.

Write your answer below.

Question 4. (15 points)

The universe of structure \mathcal{A} is $\{3, 4, 5, 6\}$. The \mathcal{A} interpretations of a , x , and y are $a^{\mathcal{A}} = 6$, $x^{\mathcal{A}} = 3$, $y^{\mathcal{A}} = 4$. The tables for the functions $f^{\mathcal{A}}$ and $g^{\mathcal{A}}$ and the predicate $P^{\mathcal{A}}$ are shown in Figure 1. Evaluate the terms

u	$f^A[u]$
3	4
4	5
5	6
6	3
	$f^A[u]$

$u \backslash v$	3	4	5	6
3	5	3	5	3
4	6	4	6	4
5	3	5	3	5
6	4	6	4	6
	$g^A[u, v]$			

$u \backslash v$	3	4	5	6
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1
6	1	1	1	1
	$P^A[u, v]$			

Figure 1: Tables for Question 4.

and formulas below. Do not show your work, just write the answer after the equal sign.

1. $\mathcal{A}[g(x, y)] =$
2. $\mathcal{A}[g(g(a, y), f(y))] =$
3. $\mathcal{A}[E(g(x, a), f(y))] =$
4. $\mathcal{A}[\forall x \exists y P(x, y)] =$
5. $\mathcal{A}[\exists x \forall y P(a, g(x, y))] =$

Question 5. (12 points)

Prove the following statement:

If x is not free in G , then $(\forall x F \longrightarrow G) \equiv \exists x (F \longrightarrow G)$.

Write your answer on a blank sheet of paper.

Question 6. (18 points)

Let t be free for x in $F \vee G$. Prove that $(F \vee G)[x/t], \forall x \neg G \models \exists x F$.

Write your answer on a blank sheet of paper.

Question 7. (12 points)

We create the data base shown below.

% printing a list

```
print([]).
print([X]):- write(X).
print([X,Y|Tail]):- write(Y), nl, print(Tail), nl, write(X).
% some relationships
loves(john,mary).
loves(john,susan).
loves(susan,john).
loves(mary,john).
```

```
% a path predicate
```

```
arc(a,b).
arc(b,c).
arc(c,a).
arc(a,d).
path(X,X).
path(X,Y):- path(X,Z),arc(Z,Y).
```

What will be printed out by the queries below?

Write the answer below the query.

```
?- print([1,2,3,4,5]).
```

```
?- path(a,d).
```

```
?- path(d,a).
```

```
?- loves(X,Y), loves(Y,X).
```

Question 8. (18 points)

a. (8 points)

Write the predicate `isIntegerList(L)` that is satisfied if L is a list of integers. Show your code below.

b. (10 points)

Write the predicate `listSum(L,S)` which is satisfied if S is the sum of the integers in the list L . Your program must ignore the items of L that are not integers. Write your program below.