

Lecture Notes for EGN 5424  
Engineering Applications of Complex Analysis  
Summer and Fall Semesters

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The current syllabus and the takehome exams are available from the course Blackboard site.

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Generic Syllabus: **EGN 5424 or EEL 6935 APPLICATIONS of COMPLEX ANALYSIS**

**Administered by Dr. Snider. Lectures are pre-taped, and delivered by web.**

**Instructor:** Prof. Dave Snider, ENB 246A, 813-9744785, FAX: 813/974-5250  
snider@eng.usf.edu . Office hours to be announced (see Blackboard Announcements).

**Catalog Description:** Analytic functions, conformal mapping, residue theory, Laurent series, transforms. Applications to various problems in engineering and physics.

**Course Prerequisites:** MAP 2302 Differential Equations (solution methods for first order nonlinear and higher order linear equations)

**Text (REQUIRED):** *FUNDAMENTALS OF COMPLEX ANALYSIS*, 3rd edition, by E B Saff & A D Snider, Prentice Hall, 2003 (ISBN 0-13-907874-6)

**Welcome to my classes, USF students!**

You're taking a "web-only" class from me. That means you won't see me live; you'll watch lectures I taped in the past, on the course materials. I will be available in my office for consultation a few hours per week, and I'll always answer your emails promptly.

Think about this: you're taking a course from a professor whom you'll hardly ever see. So you don't have much chance for an "A" unless you

(1) buy the textbook and

(2) *follow the web procedures precisely.*

**Be sure to read this long-winded syllabus entirely.** Then print it out and stick it in the back of your textbook for future reference.

You may have misgivings about non-live lectures. Let me relate a few observations that I have made over the past three years with this process.

First of all, this procedure has proved quite successful - to my surprise. I was leery about it at first, but I soon found out that the average student performance on tests was actually slightly better than it had been before, when I was lecturing live. I think the reasons for this are as follows:

1. You will never need to miss a lecture due to illness, conflicting appointments, being out of town, or simply being tired. The lectures are at the APEX web site all semester long, 24/7.
2. You can re-watch any lecture, or part of a lecture, as many times as you need.
3. If you need to take a break (no student has ever fallen asleep during my lectures, of course!!!), you can stop the playback and resume when you return.

Consult your email and my.usf.edu frequently for updates from me on assignments, test dates, and procedures in general. *Don't take any administrative instructions that I give in the lectures seriously - they are out of date. I will keep you informed about test procedures, etc. by email and my.usf.edu as the semester progresses.*

### **Two tips on calculating:**

**Some examples require a lot of (accurate) calculation. Here are two pointers that you may not have picked up yet:**

1. If you require a certain number of significant digits in a final answer, you must maintain *more* digits in the intermediate calculations. For instance consider the addition  $1/3 + 1/3 = 2/3$ . The three-significant-digit expression for the answer,  $2/3$ , is .667 . But if you first round the addends to three significant digits you get  $.333 + .333 = .666$ , which is *not correct* to 3 digits. A good rule of thumb is to retain at least two more digits than required, in *all* intermediate calculations.

2. Always introduce symbols (letters) when you evaluate a formula, especially on a calculator. For instance suppose you wanted to add 436 and 578 on a calculator. The worst way to do it is to enter 436, press enter, press +, enter 578, press enter, and press =. Because: suppose you get an answer that you know is wrong, like -23. You don't know whether you entered the 436 wrong, or the 578, or the + sign. You have to reenter all the data again. The smart way is to let A equal 436, let B equal 578, and call for A+B. If the answer is absurd, you can recall A, B, and the formula; and you can correct only the one that's wrong. This is particularly significant when you're dealing with high-digit numbers, and complicated formulas that may require parentheses.

**Remember these tips in *all* your classes.**

**Lecture notes** replicating everything that appears on the blackboard during the lectures can be purchased from

> Pro-Copy  
> 5219 E. Fowler Ave.  
> Tampa, FL 33617  
> [procopy1@aol.com](mailto:procopy1@aol.com)<<mailto:procopy1@aol.com>>  
> (813) 988-5900

They can also be downloaded from Blackboard. (Ignore notes in the .xbk format.) If you download, print out the lecture notes before you watch the lecture, and have them in front of you.

## Requirements and Assessment:

1. Most communications between instructor and students are accessed by email or through <https://my.usf.edu>. This site contains class announcements, supplementary notes, etc.

2. The lectures can be viewed at

<http://netcast.usf.edu/browse.php?page=Classes/engineering/snider/ComplexAnalysis>

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They were taped in the summer of 2004, and you will have to figure out the right pace to get you through them during the time allotted in your current semester. If you can't do this, please drop my class and re-take MTH 101, paying particular attention to the lessons on ratio and proportion. All **administrative procedures discussed in the lectures are out of date; any and all announcements posted on my.usf.edu and/or class email take precedence over anything that is stated in the lectures. Consult my.usf.edu weekly to stay abreast of assignments, test procedures, etc.**

3. Each student must email Prof. Snider by (see Blackboard) with the following data:

Last name: \_\_\_\_\_ First name: \_\_\_\_\_ Class: EGN 5424. I will not acknowledge these emails individually. You should receive an acknowledgement from me within one week stating that your address has been stored in my class email file; if you do not receive this acknowledgement, email me again – until I acknowledge receipt. **Thereafter each student is liable for all email notices concerning the class from Prof. Snider.** Students who wish to use different email boxes should email this data from each box. **Do not attempt to use one box to request email for another box.**

**3. Each student must sign a copy of the last page of this syllabus as indicated below, and mail it (fax or scan are not acceptable) by (see Blackboard) to** Prof. A D Snider, EE Dept., University of South Florida, 4202 East Fowler Avenue ENB 118, Tampa FL 33620. (Or hand-deliver it to my mailbox in the EE Department.) I will not acknowledge receipt of these syllabi individually. They can be regenerated later in the semester if a problem arises.

4. Homework problems will be recommended to the students, but not graded.

5. All three exams are takehome. The first exam is based on chapters 1-3 (complex arithmetic, Laplace's equation). The second exam will cover chapter 7. The third exam covers chapter 6. The takehome tests are heavily computational and will be computer graded with no partial credit, but you will be allowed three attempts at each test (with, however, different numerical parameters each time). You will submit your answers by email to my TA (see Blackboard Announcements), and I will shortly respond with your score, a tabulation of your incorrect answers, and the correct answers for the parameters you used. Your first attempt is assessed at (only) 5%, the second at 60%, and the third at 35%. Each attempt has a deadline. (See Blackboard Announcements)

If you miss a deadline, your subsequent submission will also count as the missed submission. If you are satisfied with your score you may stop submitting at any time, and your last submission will count for the subsequent ones. The takehome tests will be available at my.usf.edu, and they contain more detailed instructions.

The three tests are weighted 20%, 40%, and 40% respectively; however, you must get at least 2 A's and 1 B on these exams to get an A in the class. An "incomplete" grade will be awarded if either the email, syllabus signoff, or any exam are not submitted. Check with USF administration for policy details. **Retaking** a graded test, however, requires reenrollment.

A final note: I am a retiree, and I do not plan to be at USF except for my posted office hours. If you foresee some difficulties with adhering to my schedule of tests and assignments this semester, please take my course some other time, and enroll in courses with fulltime faculty members who can better accommodate your circumstances.

**Please mail a copy of this page to Dr. Snider, and place a copy of pages 1-2 in your textbook.**

**Academic Dishonesty** - It is not acceptable to copy, plagiarize or otherwise make use of the work of others in completing homework, project, exam or other course assignments. The minimum penalty for doing so is an automatic zero on the assignment and an "F" in the course. **I have read the syllabus for EGN 5424 and agree to abide by its schedule and terms.**

(print name)

(sign name)

## Guide to the Complex Analysis Lecture Movies 1

Jun 28 Complex Arithmetic

Jun 29 Complex Plane; Polar Coordinates,  $\text{Arg } z$

Jun 30 Euler's Formula, Powers and Roots

Jul 1 LaPlace's Equation, Equilibrium temperatures

Jul 6 Thermal Interpretation of  $\text{Log } z$  ; 2-D electrostatics

Jul 7 Wedges, walls, and washers; Analyticity

Jul 8 Cauchy-Riemann equations, harmonic functions

Jul 12 Complex Transcendental Functions; Isotherms 1

Jul 13 Complex Integrals; Isotherms 2

Jul 14 Contours, Cauchy Integral Theorem

Jul 15 Cauchy Integral Formula; Taylor's Series; Conformal Map

Jul 19 Cauchy Integral Formula; Conformal Map

Jul 20: Maximum Principle; Liouville Theorem; Fundamental Theorems; Moebius Transformations

Jul 21 Taylor Series, Moebius Cross-Ratio

Jul 21a Smith Chart (Notes appear separately)

Jul 22 Moebius Transformations

Jul 26 Conformality; Schwarz-Christoffel transformation

July 27 Laurent Series

July 29 Residue Theory; Jordan Lemma

Aug 2 Jordan lemma; Principle Value of Integrals; Real poles

Aug 3 Multiple-Valued Integrands

Aug 4 Singularities; Integration Examples

Aug 5 Principle of the Argument; Rouché Theorem

Guide to the Complex Analysis Lecture Movies II

EGN 5424

RENAME

<http://netcast.usf.edu/browse.php?page=Classes/engineering/snider/egn5424>

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<http://netcast.usf.edu/browse.php?page=Classes/engineering/snider/ComplexAnalysis>

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[egn5424ca\\_Lec1\\_Arithmetic](#)  
[egn5424ca\\_Lec2\\_ComplexPlane](#)  
[egn5424ca\\_Lec3\\_Euler\\_Roots](#)  
[egn5424ca\\_Lec4\\_LaplaceEq](#)  
[egn5424ca\\_Lec5\\_Log\\_Electrostatics](#)  
[egn5424ca\\_Lec6\\_Analyticity](#)  
[egn5424ca\\_Lec7\\_CauchyRiemann](#)  
[egn5424ca\\_Lec8\\_TranscendentalFcns](#)  
[egn5424ca\\_Lec9\\_Integrals](#)  
[egn5424ca\\_Lec10\\_Contours](#)  
[egn5424ca\\_Lec11\\_CauchyTheorem](#)  
[egn5424ca\\_Lec12\\_Taylor\\_Conformal](#)  
[egn5424ca\\_Lec13\\_MaxPrinc\\_Moebius](#)  
[egn5424ca\\_Lec14\\_Taylor\\_Moebius](#)  
[egn5424ca\\_Lec15\\_SmithChart](#)  
[egn5424ca\\_Lec16\\_Moebius](#)  
[egn5424ca\\_Lec17\\_SchwarzChristoffel](#)  
[egn5424ca\\_Lec18\\_LaurentSeries](#)  
[egn5424ca\\_Lec19Residues\\_Jordan](#)  
[egn5424ca\\_Lec20\\_PV\\_RealPoles](#)  
[egn5424ca\\_Lec21\\_MultivaluedInts](#)  
[egn5424ca\\_Lec22\\_Singularities](#)  
[egn5424ca\\_Lec23\\_ArgPrin\\_Rouche](#)

Some random notes that are cited in the lectures

Section 1.5 #19 (page 38)

Look at Fig. 1.13 (page 22). If a + particle is at the origin and a unit test charge is at  $z$ , then  $1/\bar{z}$  points toward  $z$ , in the direction of the repulsive force; moreover, it has the right magnitude ( $1/|z|$ ). To get the force on  $z$  from a unit charge situated at  $z_1$ , use the same formula with  $z$  measured *relative to*  $z_1$ :  $1/(\overline{z - z_1})$ .

The total force on  $z$  from unit charges at  $z_1, z_2$ , and  $z_3$  is  $1/(\overline{z - z_1}) + 1/(\overline{z - z_2}) + 1/(\overline{z - z_3})$ .

At equilibrium,

$$\frac{1}{z - z_1} + \frac{1}{z - z_2} + \frac{1}{z - z_3} = 0. \text{ Common denominators implies}$$
$$(\overline{z - z_2})(\overline{z - z_3}) + (\overline{z - z_1})(\overline{z - z_3}) + (\overline{z - z_1})(\overline{z - z_2}) = 0.$$

With  $z_1 = 1+i, z_2 = -1+i, z_3 = 0$ , this gives

$$3\bar{z}^2 + 4i\bar{z} - 2 = 0. \text{ From the quadratic formula } \bar{z} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ with } a=3, b=4i,$$

$c=-2$ :

$$\bar{z} = \frac{-4i \pm \sqrt{-16 + 24}}{6} = \frac{-4i \pm \sqrt{8}}{6} = \pm \frac{\sqrt{2} - 2i}{3}, \text{ so } z = \pm \frac{\sqrt{2} + 2i}{3}.$$



Suggested problems for practice in conformal mapping.

Sec. 7.1 #3

Sec. 7.3 #1, 2, 3, 4, 5, 8, 9, 11, 12

Sec. 7.4 #1, 2, 6

Sec. 7.5 #2, 3, 4, 7, 9 if you have time

Sec. 7.6 #1, 2, 4, 6, 7, 8, 9, 10

Answers to even-numbered problems in Section 7.6.

$$2. \phi = 1 - \frac{2}{\pi} \operatorname{Arg} \left\{ -i \frac{1-z}{1+z} \right\}$$

$$4. \tau = 3 - \frac{1}{\pi} \left[ \operatorname{Arg} \left\{ i \frac{1+z}{1-z} + 1 \right\} + \operatorname{Arg} \left\{ i \frac{1+z}{1-z} \right\} + \operatorname{Arg} \left\{ i \frac{1+z}{1-z} - 1 \right\} \right]$$

$$6. \phi = 2 \operatorname{Re} \frac{z}{z-2}$$

$$8. \phi = \frac{1}{\operatorname{Log}(5+2\sqrt{6})} \operatorname{Log} \left| \frac{(7+2\sqrt{6})z - 11 - 6\sqrt{6}}{5z - 1 + 4\sqrt{6}} \right|$$

$$10. \tau = \frac{6}{\pi} \operatorname{Arg} \left( \frac{z - e^{i5\pi/6}}{z - e^{i\pi/6}} \right) + 3$$

## Accurate calculations and evaluation of integrals using MATLAB.

### Two tips on calculating:

**Some tests in this course require a lot of calculation. Here are two pointers that you may not have picked up yet:**

1. If you require a certain number of significant digits in a final answer, you must maintain *more* digits in the intermediate calculations. For instance consider the addition  $1/3 + 1/3 = 2/3$ . The three-significant-digit expression for the answer,  $2/3$ , is .667 . But if you first round the addends to three significant digits you get  $.333 + .333 = .666$ , which is *not correct* to 3 digits. A good rule of thumb is to retain at least two more digits than required, in *all* intermediate calculations.

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**Remember these tips in all your classes.**

Note: if you don't know MATLAB, you still should be able to use it to evaluate integrals; just open it (click on the MATLAB icon) and notice how functions are entered into the "@" statements below - multiplication, sines, exponentials, division, CORRECT NUMBER OF PARENTHESES.

I. **One-dimensional integrals.** To integrate  $\int_{1.234}^{5.678} [e^{-2x^2} \cos(4x + 7) - \frac{2x + 1}{(x + 3)^2 (x + 1)(x + 2)}] dx$

:

in MATLAB type

format long (Enter)

a=2 (Enter)

b=4 (Enter)

c=7 (Enter)

d=2 (Enter)

e=1 (Enter)

f=3 (Enter)

g=1 (Enter)

h=2 (Enter)

lower=1.234 (Enter)

upper=5.678 (Enter)

accuracy = 1e-8 (Enter) (Of course you may want more or less accuracy.)

Now enter the formula for the integrand, REGARDING THE VARIABLE OF INTEGRATION (x) AS A MATRIX. That means inserting the period mark (.) in front of all multiplication signs \*, division signs /, and exponentiation carats ^ involving x.

integrand=@(x)(exp(-a\*x.^2).\*cos(b.\*x+c) - (d.\*x+e)./((x+f).^2.\*(x+g).\*(x+h))) (Enter)

(You'll probably get some typo error message here; retype carefully, counting parentheses and inserting dots. An error message like

"???" Error using ==> mtimes

Inner matrix dimensions must agree."

means you left out some dots. An error message like

" Error: Expression or statement is incorrect--possibly unbalanced (, {, or [."

usually means your parentheses are wrong. *If you change the values of any of the parameters, reenter the integrand=@... statement.*) Now type

Q=quadl(integrand, lower, upper, accuracy) (Enter)

I got an answer of Q = -0.037041299610442 .

II. **Two-dimensional integrals.** To integrate  $\int_4^5 \int_6^7 (u^2 + uv) dv du$  :

in MATLAB type

format long (Enter)

lowerv=6 (Enter)

upperv=7 (Enter)

loweru=4 (Enter)

upperu=5 (Enter)

accuracy = 1e-8 (Enter)

integrand=@(u,v)(u.^2+u.\*v) (Enter)

Q= dblquad(integrand, loweru,upperu,lowerv,upperv, accuracy, @quadl) (Enter)

I got Q = 49.58333333333329

III. **Three-dimensional integrals.** To integrate  $\int_2^3 \int_4^5 \int_6^7 uvw dw dv du$  :

in MATLAB type

format long (Enter)

loww=6 (Enter)

highw=7 (Enter)

lowv=4 (Enter)

highv=5 (Enter)

lowu=2 (Enter)

highu=3 (Enter)

accuracy = 1e-8 (Enter)

integrand=@(u,v,w)(u.\*v.\*w) (Enter)

Q= triplequad(integrand, lowu,highu,lowv,highv,loww,highw,accuracy, @quadl) (Enter)

I got Q = 73.12499999999986 .

To be conservative, try out your code with integrands that you can do by hand, to check.