COT 5407 Introduction to Algorithms

Homework 1

DUE: Thursday, September 9, 2010

Please remember that all submissions must be typeset. Handwritten submissions will NOT be accepted.

1. Order the following function by growth rate. N, $\sqrt{N}$, $N^{1.5}$, N^2, $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, 2^N, $2^{N/2}$, 37, $N^2 \log N$, N^3. Indicate which functions grow at the same rate.

2. An algorithm takes 0.5 ms for input size 100. How long will it take for input size 500 if the running time is the following (asume low-order terms are negligible):

   (a) linear
   (b) $O(N \log N)$
   (c) quadratic
   (d) cubic

3. An algorithm takes 0.5 ms for input size 100. How large a problem can be solved in one minute if the running time is the following (asume low-order terms are negligible):

   (a) linear
   (b) $O(N \log N)$
   (c) quadratic
   (d) cubic

4. Solve the following equations, expressing the answer in Big-Oh notation, Assume that $T(N)$ is constant for sufficiently small N.

   (a) $T(N) = T(N/2) + 1$
   (b) $T(N) = T(N-1) + \log N$
   (c) $T(N) = T(N-1) + N$
   (d) $T(N) = 8T(N/2) + N^2$
   (e) $T(N) = 8T(N/2) + N^3$
   (f) $T(N) = 8T(N/2) + N^4$

5. *Three-way partitioning* is a modification of quicksort that partitions elements into groups smaller than, equal to, and larger than the pivot. Only the groups of smaller and larger elements need to be recursively sorted. Show that if there are N items but only k unique values (in other words there are many duplicates), then the running time of this modification to quicksort is $O(Nk)$.

6. Consider the following algorithm for sorting six numbers:

   - Sort the first three numbers using Algorithm A.
   - Sort the second three numbers using Algorithm B.
   - Merge the two sorted groups using Algorithm C.

   Show that this algorithm is suboptimal, regardless of the choices for Algorithms A, B, and C.

7. You are given N numbers (some of which are negative). Give an $O(N^2 \log N)$ algorithm to decide if there are four numbers in the group that sum to exactly 0. Numbers may be used more than once to form the sum.