

COT 5407 Introduction to Algorithms

Homework 4

DUE: Tuesday, November 2, 2010

Please remember that all submissions are to be your own work and must be typeset. Handwritten submissions will NOT be accepted.

1. Show that if all the **unions** precede the **finds**, then the disjoint set algorithm with path compression and **union** by rank requires linear time. Note, that this question does not ask you to design a new algorithm; it asks you to show a better time bound for the algorithm in class, for a special (simpler) set of input.
2. When a vertex and its incident edges are removed from an undirected tree, a collection of subtrees remains. Give a linear-time algorithm that finds a vertex whose removal from an N -vertex tree leaves no subtree with more than $N/2$ vertices.
3. Do CLRS Problem 23-4, page 641.
4. The capacity flow along a path in a directed graph is the value of the minimum edge on the path (for zero length paths, the capacity flow is infinite). Describe (with running time analysis) an efficient algorithm that given two vertices s and t , finds the path from s to t with the maximum capacity flow (if there is no path from s to t , the maximum capacity flow is 0).
5. Let $G = (V, E)$ be an undirected graph. Use depth-first search to design a linear-time algorithm to convert each edge in G to a directed edge such that the resulting graph is strongly connected, or determine that this is not possible.