COT 5407 Introduction to Algorithms

Homework 5

DUE: Tuesday, November 30, 2010

I would like to strongly reiterate that all submissions are to be your own work and must be typeset. Joint work and work that does not properly cite sources will recieve no credit. Handwritten submissions will NOT be accepted.

1. What is the optimal way to compute $A_1A_2A_3A_4A_5A_6$, where the dimensions of the matrices are A_1: 10 x 20, A_2: 20 x 5, A_3: 5 x 40, A_4: 40 x 5, A_5: 5 x 60, A_6: 60 x 15?

2. Let A be an N-by-N matrix of zeros and ones. A submatrix S of A is any group of *contiguous* entries that form a square. Design an $O(N^2)$ algorithm that determines the size of the largest submatrix of ones in A. For instance, in the matrix that follows, the largest submatrix is a 4-by-4 square.

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10111000
00101000
00111000
00111010
00111111
01011110
01011111
00011110
```

3. Given an *undirected* graph $G = (V, E)$, and an integer K, the **ARC-DELETION** problem is that of determining if there is a set of K edges whose deletion breaks all cycles. Either give a polynomial-time algorithm or prove that ARC-DELETION is NP-complete.

4. A coin collector has an opportunity to purchase N coin collections $S_1, S_2, ..., S_N$. Some collections may contain coins in common with others. The collector wants to buy at least K collections out of the N, but **must avoid duplicates**. The **COIN COLLECTOR'S PROBLEM** is whether there is a selection of at least K mutually disjoint collections out of the N. Either give a polynomial-time algorithm or prove that COIN COLLECTOR'S PROBLEM is NP-complete.

5. CLRS Problem 34-2 (Bonnie and Clyde), page 1102.

6. For the vertex cover problem, suppose the graph is acylic. Either prove that under this restriction vertex cover remains NP-complete, or provide a polynomial-time algorithm.

7. For the bin packing problem, suppose all item sizes are **greater than** 1/3. Either prove that under this restriction bin packing remains NP-complete, or provide a polynomial-time algorithm.