COT 5407 Introduction to Algorithms

Homework 4

DUE: Tuesday, November 8, 2011

Please remember that all submissions are to be your own work and must be typeset. Handwritten submissions will NOT be accepted.

1. For union-by-rank with path compression, show
   (a) $C(M, N, r)$ is 0 if $r \leq 1$.
   (b) $C(M, N, 2)$ is at most M.
   (c) $C(M, N, r) \leq M + N$ if $r \leq 8$ (hint: choose an appropriate s).
2. When a vertex and its incident edges are removed from an undirected tree, a collection of subtrees remains. Give a linear-time algorithm that finds a vertex whose removal from an N-vertex tree leaves no subtree with more than $N/2$ vertices.
3. Integers $1, 2, ..., E$ are each randomly assigned as a weight to an edge in an undirected graph with E edges and V vertices. Give an $O(E\alpha(E, V))$ algorithm to find the minimum spanning tree of this graph.
4. Give a linear-time algorithm to find the **longest weighted path** from s to t in a directed **ACYCLIC** graph.
5. Let $G = (V, E)$ be an undirected graph. Use depth-first search to design a linear-time algorithm to convert each edge in G to a directed edge such that the resulting graph is strongly connected, or determine that this is not possible.