1. For union-by-rank with path compression, show
   (a) \( C(M, N, r) \) is 0 if \( r \leq 1 \).
   (b) \( C(M, N, 2) \) is at most \( M \).
   (c) \( C(M, N, r) \leq M + N \) if \( r \leq 8 \) (hint: choose an appropriate \( s \)).

2. When a vertex and its incident edges are removed from an undirected tree, a collection of subtrees remains. Give a linear-time algorithm that finds a vertex whose removal from an \( N \)-vertex tree leaves no subtree with more than \( N/2 \) vertices.

3. Integers 1, 2, ..., \( E \) are each randomly assigned as a weight to an edge in an undirected graph with \( E \) edges and \( V \) vertices. Give an \( O(E\alpha(E, V)) \) algorithm to find the minimum spanning tree of this graph.

4. Give a linear-time algorithm to find the longest weighted path from \( s \) to \( t \) in a directed ACYCLIC graph.

5. Let \( G = (V, E) \) be an undirected graph. Use depth-first search to design a linear-time algorithm to convert each edge in \( G \) to a directed edge such that the resulting graph is strongly connected, or determine that this is not possible.