In a proof by induction, the *basis* is the easy case that can be shown by hand.

The *inductive hypothesis* assumes that the theorem is true for some arbitrary case and shows that, under this assumption, it is true for the next case.

Why does this constitute a proof? First, the theorem is true for $N = 1$. This is called the *basis*. One can view it as being the basis for our belief that the theorem is true in general. The basis is the easy case(s) that can be shown by hand. Once we have established the basis, we can hypothesize that the theorem is true for some arbitrary k. This is called the *inductive hypothesis*. We show that if the theorem is true for k, then it is true for $k + 1$. In our case, since we know the theorem is true for the basis $N = 1$, we know that it is true for $N = 2$. Since it is true for $N = 2$, it must be true for $N = 3$. And since it is true for $N = 3$, it must be true for $N = 4$. Extending this logic, we see that the theorem is true for every positive integer beginning with $N = 1$.

Let us apply proof by induction to a second problem, one that is not quite as simple as the first. First, we examine the sequence of numbers 1^2, $2^2 - 1^2$, $3^2 - 2^2 + 1^2$, $4^2 - 3^2 + 2^2 - 1^2$, $5^2 - 4^2 + 3^2 - 2^2 + 1^2$, and so on. Each member represents the sum of the first N squares, with alternating signs. The sequence evaluates to 1, 3, 6, 10, and 15. It seems that, in general, the sum is exactly equal to the sum of the first N integers, which, as we know from Theorem 7.1, would be $N(N + 1)/2$. Theorem 7.2 proves this.

Theorem 7.2 *The sum* $\sum_{i=N}^{1}(-1)^{N-i}i^2 = N^2 - (N-1)^2 + (N-2)^2 \ldots$ *is* $N(N+1)/2$.

Proof *The proof is by induction.*

Basis: Clearly, the theorem is true for $N = 1$.

Inductive hypothesis: First, we assume the theorem is true for k: $\sum_{i=k}^{1}(-1)^{k-i}i^2 = \frac{k(k+1)}{2}$. *Then, we must show that it is true for* $k + 1$*; namely, that* $\sum_{i=k+1}^{1}(-1)^{k+1-i}i^2 = \frac{(k+1)(k+2)}{2}$. *We write*

$$\sum_{i=k+1}^{1}(-1)^{k+1-i}i^2 = (k+1)^2 - k^2 + (k-1)^2 \ldots . \tag{7.3}$$

If we rewrite the right-hand side of Equation 7.3, we obtain

$$\sum_{i=k+1}^{1}(-1)^{k+1-i}i^2 = (k+1)^2 - (k^2 - (k-1)^2 \ldots).$$

This allows a substitution to yield