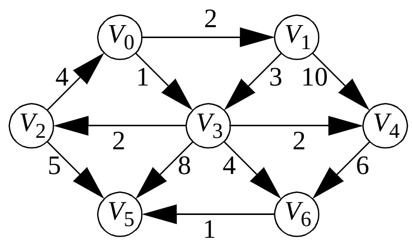

Figure 14.1 A directed graph

The following vertices are adjacent to V_3: V_2, V_4, V_5, V_6. For this graph, $|V| = 7$ and $|E| = 12$; here, $|S|$ represents the size of set S.

A *path* is a sequence of vertices that are connected by edges.

A *path* in a graph is a sequence of vertices $w_1, w_2, \ldots, w_N$ such that $(w_i, w_{i+1}) \in E$ for $1 \le i < N$. The *length* of such a path is the number of edges on the path, namely, $N - 1$. This is called the *unweighted path length*. The *weighted path length* is the sum of the costs of the edges on the path. As an example, V_0, V_3, V_5 is a path from vertex V_0 to V_5. The path length is two edges, and the weighted path length is 9 — this is the shortest path between V_0 and V_5. However, if the cost is important, then the weighted shortest path between these vertices has cost 6 and is V_0, V_3, V_6, V_5. A path may exist from a vertex to itself. If this path contains no edges, then the path length is 0. This is a convenient way to define an otherwise special case. A *simple path* is a path in which all vertices are distinct, except that the first and last can be the same.

A *cycle* in a directed graph is a path that begins and ends at the same vertex and contains at least one edge.

A *cycle* in a directed graph is a path of length at least 1 such that $w_1 = w_N$; this cycle is simple if the path is simple. A *directed acyclic graph*, sometimes referred to by its abbreviation, *DAG*, is a directed graph with no cycles.

An example of a real-life situation that can be modeled by a graph is the airport system. Each airport is a vertex. If there is a nonstop flight between the corresponding airports, two vertices are connected by an edge. The edge could have a weight, representing time, distance, or the cost of the flight. Generally, an edge (v, w) would imply an edge (w, v). But it is reasonable to assume that the costs of the edges might be different, since flying in different directions might take longer (depending on prevailing winds) or cost more (depending on local taxes). Naturally, we want to quickly determine the best flight between any two airports; “best” could mean the path with the fewest number of edges or could be taken with respect to one, or all, of the weight measures (distance, cost, and so on).

A second example of a real-life situtation that can be modeled by a graph is the routing of electronic mail through computer networks. Vertices represent computers, the edges represent links between pairs of computers, and the edge costs represent communication costs (phone bills per megabytes), delay costs (seconds per megabyte), or combinations of these and other factors.