



**Figure 21.7** Result of splaying at node 1 (three zig-zigs)

## 21.3 Basic Splay Tree Operations

As mentioned earlier, a splay operation is performed after each access. When an insertion is performed, we perform a splay. As a result, the newly inserted item becomes the root of the tree. Otherwise, we could spend quadratic time constructing an  $N$  item tree.

For the `find`, we splay at the last node that is accessed during the search. If the search is successful, then the node that is found will be splayed and become the new root. If the search is unsuccessful, then the last node that was accessed prior to reaching the null reference is splayed and becomes the new root. This behavior is necessary. Otherwise, we could repeatedly perform a `find` for 0 in the initial tree in Figure 21.7 and use linear time per operation. Likewise, operations such as `findMin` and `findMax` perform a splay after the access.

The interesting operations are the deletions. Recall that the `deleteMin` and `deleteMax` are important priority queue operations. With splay trees, these operations become simple. We can implement the `deleteMin` as follows. First, perform a `findMin`. This brings the minimum item to the root, and by the binary search tree property, there is no left child. We can use the right child as the new root. Similarly, `deleteMax` can be implemented by calling `findMax` and setting the root to the post-splay root's left child.

Even the `remove` operation is simple. To perform deletion, we access the node that is to be deleted. This puts the node at the root. If it is deleted, we get two subtrees  $L$  and  $R$  (left and right). If we find the largest element in  $L$  using a `findMax` operation, then its largest element will be rotated to  $L$ 's root and  $L$ 's root will have no right child. We finish the `remove` by making  $R$  the right child of  $L$ 's root. An example of the `remove` operation is shown in Figure 21.8 (page 624).

After an item is inserted as a leaf, it is splayed to the root.

All searching operations incorporate a splay.

Deletion operations are much simpler than usual. They also incorporate a splaying step (sometimes two).