Celebrity Problem

- A Celebrity is one that knows nobody and that everybody knows.

Celebrity Problem:
INPUT: \(n\) persons with a \(n \times n\) information matrix.
OUTPUT: Find the “celebrity”, if one exists.
MODEL: Only allowable questions are:
- Does person \(i\) know person \(j\)?

- Naive Algorithm: \(O(n^2)\) Questions.
Celebrity Problem (Cont’d)

- **Induction Hypothesis:** We know how to find a celebrity (if one exists) among a set of \( n-1 \) people.

[The above hypothesis leads to inefficient solution.]

Given \( n \) persons, 3 cases arise:

1. Celebrity is among the first \( n-1 \) persons
2. Celebrity is the \( n \)-th person.
3. No celebrity exists.
Celebrity Problem (Cont’d)

– **Induction Hypothesis 2:** We know how to find $n-2$ non-celebrities among a set of $n-1$ people, i.e., we know how to find at most one person among a set of $n-1$ people that could potentially be a celebrity.

– Resulting algorithm needs $[3(n-1)-1]$ questions.
Psychic Assist Hotline

• Ms. Cleo gives me 15 numbers and promises me that at least 4 will appear in Saturday’s FL Lottery.

• How many tickets do I need to buy to guarantee at least one ticket to have 3 correct numbers?
Smaller Problem

• Suppose Ms. Cleo gives me 6 numbers from which 4 are guaranteed. Every ticket has 3 numbers and I need 3 to win.

• Cover all 3-sets.

• Suppose I pick \{1,2,3\}, \{1,2,4\} & \{1,2,5\}. Then should I also have picked \{1,2,6\}?

• NO!
Psychic Assist Hotline

• Ms. Cleo gives me 15 numbers and promises me that at least 4 will appear in Saturday’s FL Lottery.

• How many tickets do I need to buy to guarantee at least one ticket with at least 3 correct numbers?

• FIVE!!! (if you assume that numbers come from 1 through 44).
Psychic Problem

- Initialize all k-sets as “uncovered”.
- While (there is a “uncovered” k-set)
  - Select a ticket that contains it
  - Update the set of “covered” k-sets.