1. [15 points] Assuming that sets $A$, $B$, and $A \cap B$ have $m$, $n$, and $k$ elements respectively, how many elements are in the following sets?

- Power set of $A \times B$
- $(A \cup B)^2$
- $A^2 \cup B^3$
- $A^2 \cap B^3$
- Power set of $(A \cap (A^2 \times B^3))$

2. [10 points] Solve Sipser (2nd edition) 0.6, 0.7

3. [25 points]

   For the alphabet $\Sigma = \{a, b\}$,

   (a) [15 points] For the alphabet $\Sigma = \{a, b\}$, construct an NFA that recognizes the set

   \[ \{ w \in \Sigma^* : \left( 3 \left| \left( n_a(w) - n_b(w) \right) \right| \right) \lor \left( \text{aba is a substring of } w \right) \}, \]

   where $n_a : \Sigma^* \mapsto \mathbb{Z}_{\geq 0}$ and $n_b : \Sigma^* \mapsto \mathbb{Z}_{\geq 0}$ respectively count the number of $a$'s and $b$'s participating in the input string ($a | b$ if $b$ is divisible by $a$).

   (b) [10 points] Convert the obtained NFA in part (a) into an equivalent DFA (you do not have to depict states that are unreachable from the initial state).
4. [20 points]

Assuming that \( A \) and \( B \) are two regular languages,

(a) **[10 points]** Prove that \( \bar{A} \) is regular too (closure of regular languages under complementation)

(b) **[10 points]** Use Theorems 1.45, 1.47, and 1.49 of Sipser book to prove that \((A \cap B) \circ (A \setminus B^*)\) is regular \((A \setminus B = A \cap \bar{B})\).

5. [10 points]

For the DFA above do the following:

(a) Describe the language the DFA accepts as clearly and simply as you can, for example, using one short sentence.

(b) Using formal notation describe all five important pieces of the DFA.

6. [20 points] Solve Sipser (2nd edition) 1.6 b, 1.6 d, 1.5 c, 1.4 c