1. [10 points] For the following items, answer them with True or False. Please use the same notation as Sipser's book. \{\ldots\} represent sets, \( \mathbb{N} = \{1, 2, \ldots\} \) is the set of natural numbers, \( \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \) is the set of integers, and \( \mathbb{P} = \{2, 3, 5, \ldots\} \). As in the book, the symbol \( P(S) \) denotes the set of all subsets of \( S \) (the power set of \( S \)).

(a) \( \mathbb{P} \in P(\mathbb{N}) \)
(b) \( \mathbb{Z} \cup \mathbb{N} \subset \mathbb{Z} \)
(c) \( (\mathbb{Z} \setminus \mathbb{P}) \cup \mathbb{N} = (\mathbb{Z} \setminus \mathbb{N}) \cup \mathbb{P} \)
(d) \( \{\mathbb{N} \cap \{-1\}\} \in \{\emptyset\} \)
(e) \( \emptyset \in \{\{\emptyset\}, \mathbb{N}\} \)
(f) \( \emptyset \subseteq \{\emptyset\}, \mathbb{N}\} \)
(g) \( |P(\emptyset) \setminus \{\emptyset\}| = 1 \)
(h) \( \emptyset = \mathbb{N}^2 \setminus ((\mathbb{N} \cap \mathbb{Z}) \times (\mathbb{N} \cup \mathbb{P})) \)
(i) \( P(\mathbb{Z}) \times P(\mathbb{N}) \neq P(\mathbb{Z} \times \mathbb{N}) \)
(j) \( \{w \mid w = w_0 w_1 \ldots w_n, w_i \in \mathbb{Z}\} \) is a language.

2. [15 points] For the alphabet construct a DFAs that recognizes the following languages:

- The set of strings \( w \) in \( \Sigma^* \) such that \( w \) has exactly two \( a \)'s and an even number of \( b \)'s, for \( \Sigma = \{a, b\} \).
- \( A = \{w \in \Sigma^* \mid bbb \) happens exactly once as a substring in \( w\}, \) in which \( \Sigma = \{a, b\}. \) In this case \( bbbb \notin A \).
- \( \{w \in \Sigma^* \mid w \) is representing a binary number divisible by \( 4\}, \) in which \( \Sigma = \{0, 1\} \)

3. [10 points] Convert the NFA shown below into an equivalent DFA, the language is \( \Sigma = \{0, 1\} \). Put a label to each state of the DFA to indicate which states of the NFA it corresponds to. You simply need to draw a diagram for the DFA, you do not have to depict states that are unreachable from the initial state.

![Diagram of NFA](image-url)
4. **[10 points]** Find an equivalent NFA for the following this regular expression: \((b \cup a)^* (ba \cup b)\)

5. **[10 points]** Convert the DFA below into a regular expression that describes exactly the same language. Eliminate states in the following order: \(q_3, q_1, q_2\). Show your work.

![DFA Diagram]

6. **[10 points]** Prove that every finite language is regular.

7. **[15 points]** Are the following languages regular? (Prove your answers)
   - \(C_1 = \{a^p b^q a^{p+q} \in \Sigma^* \mid p \geq 0, q \geq 0\}\)
   - \(C_2 = \{a^{n^2} \in \Sigma^* \mid n \geq 2\}\)
   - \(A = \{0^n | n \geq 0\}\)

8. **[10 points]** Let \(A_1, A_2,\) and \(A_3\) be languages defined over an alphabet \(\Sigma\). Define

\[
\text{Majority}(A_1, A_2, A_3) = \{w \mid w \text{ is in at least two of } \{A_1, A_2, A_3\}\}.
\]

Show that \(\text{Majority}(A_1, A_2, A_3)\) is regular if \(A_1, A_2,\) and \(A_3\) are regular.

9. **[10 points]** Prove that regular languages are closed operations defined below:
   - \(i(A) = \{x^R \mid x \in A\}\)
   - \(C(A_1, A_2) = \{w \mid wx \in A_1 \text{ and } x \in A_2\}\)